Theory of turbo machinery / Turbomaskinernas teori

Dixon, chapter 7

Centrifugal Pumps, Fans and Compressors

And to thy speed add wings.
(MILTON, Paradise Lost.)

Centrifugal Pumps, Fans and Compressors

- **What do radial machines look like?**
- **Swept wings or not?**
- **Slip (deviation).**
- **Example.**

Material adopted from:

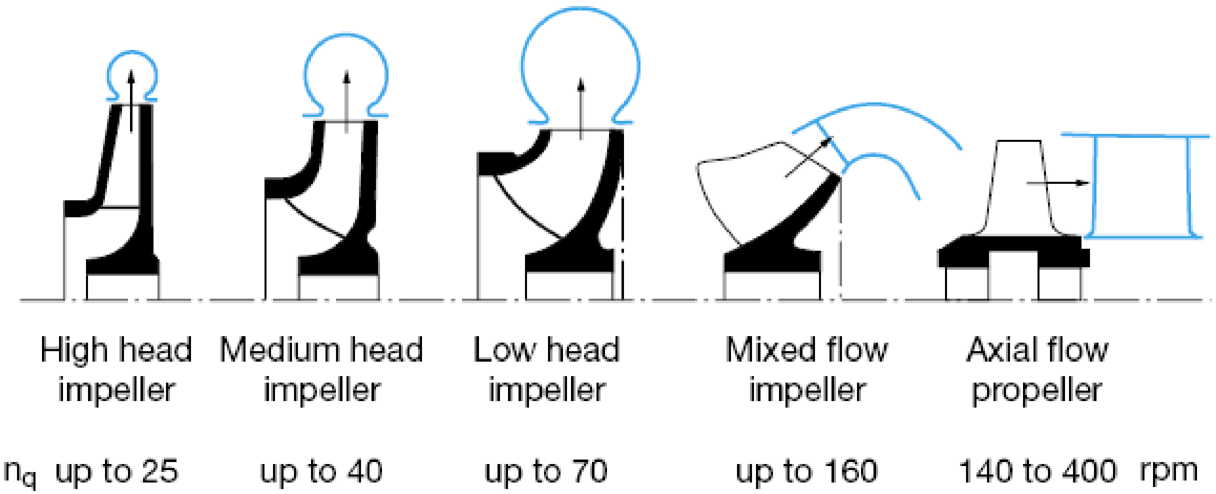
- **Alvarez, Energiteknik**
- **KSB: Selecting centrifugal pumps**

Centrifugal Pumps, Fans and Compressors

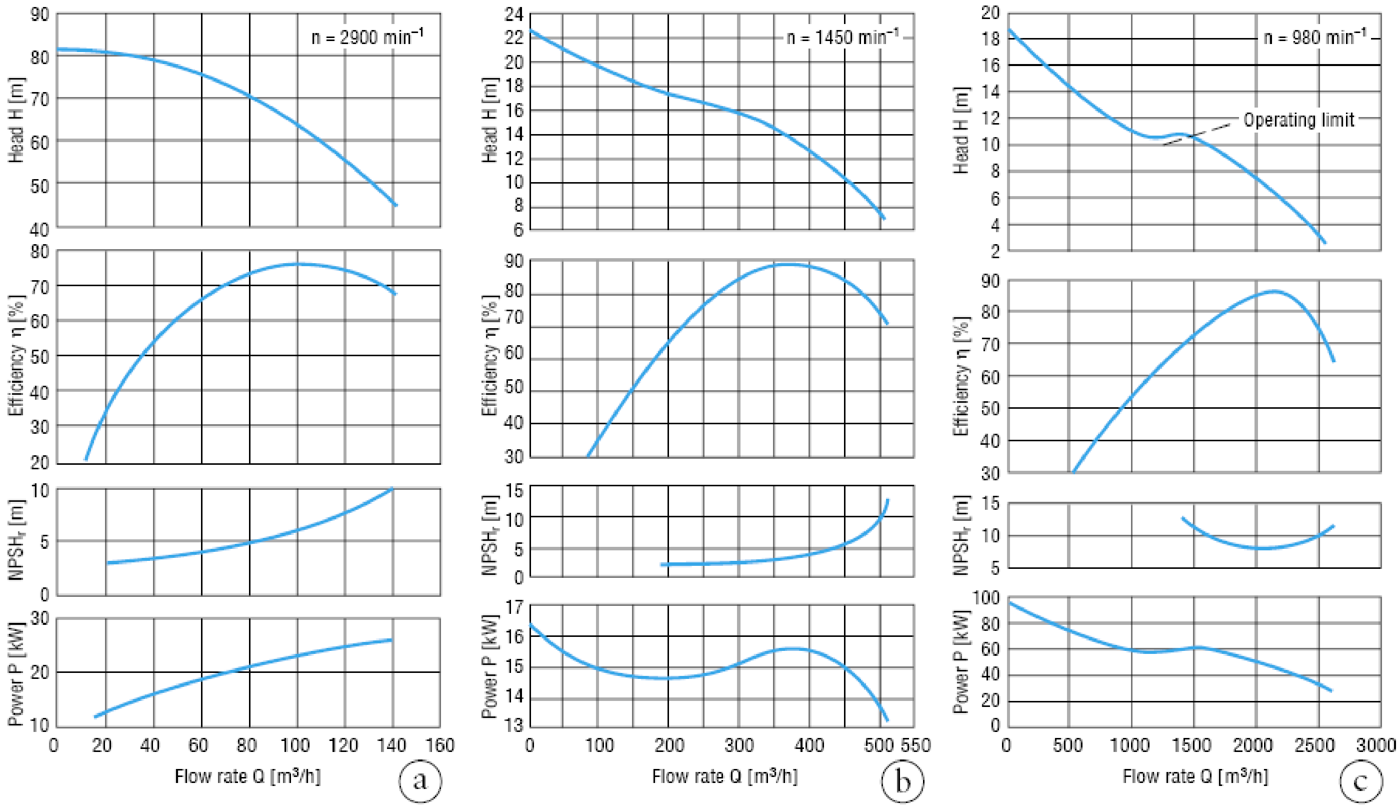
Specific speed:

- Dimensionless flow to head ratio so that *D* is eliminated.
- Values of flow and head at max efficiency

$$\boxed{N_s = \frac{\phi_1^{1/2}}{\psi_1^{3/4}} = \frac{NQ^{1/2}}{(gH)^{3/4}}}$$

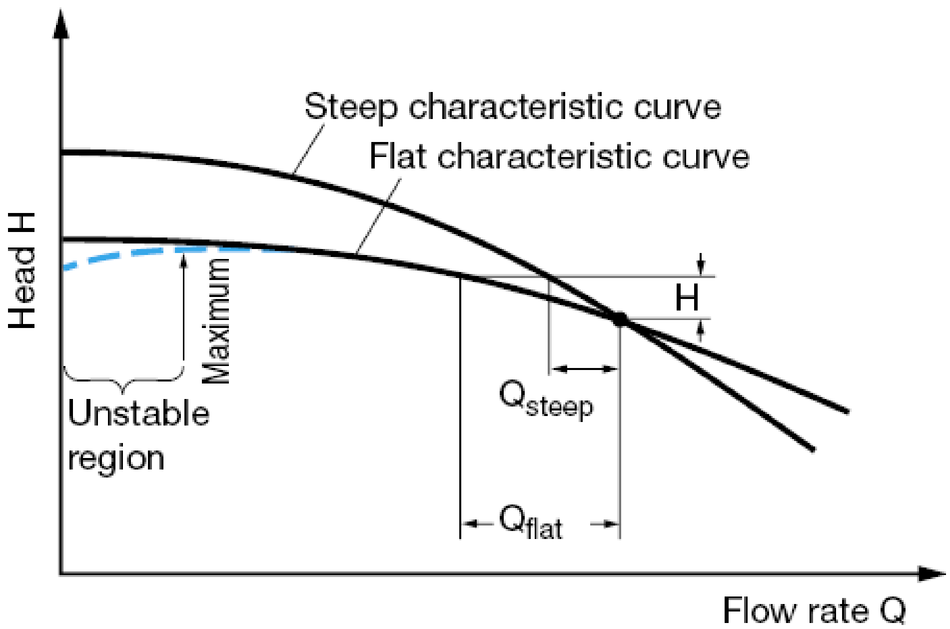


Centrifugal Pumps, Fans and Compressors



Three examples of characteristic curves for pumps of differing specific speeds. a: radial impeller, $n_q \approx 20$; b: mixed flow impeller, $n_q \approx 80$; c: axial flow impeller, $n_q \approx 200$.

Centrifugal Pumps, Fans and Compressors

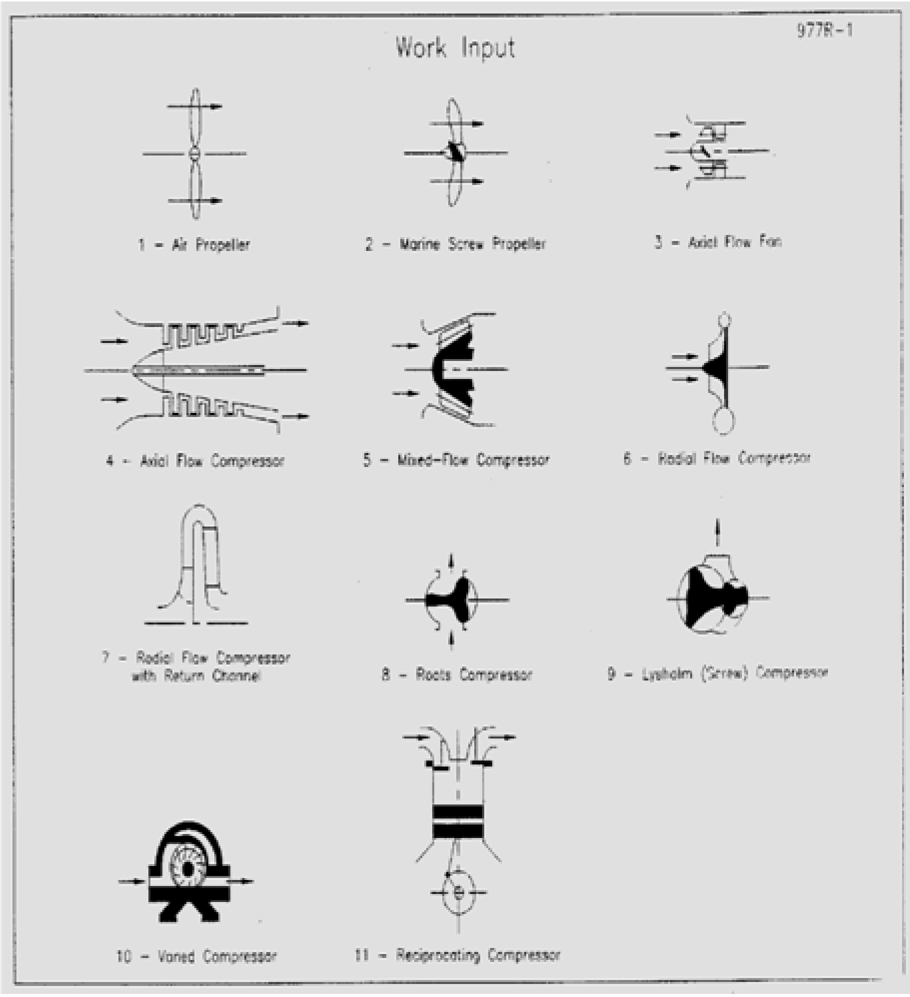


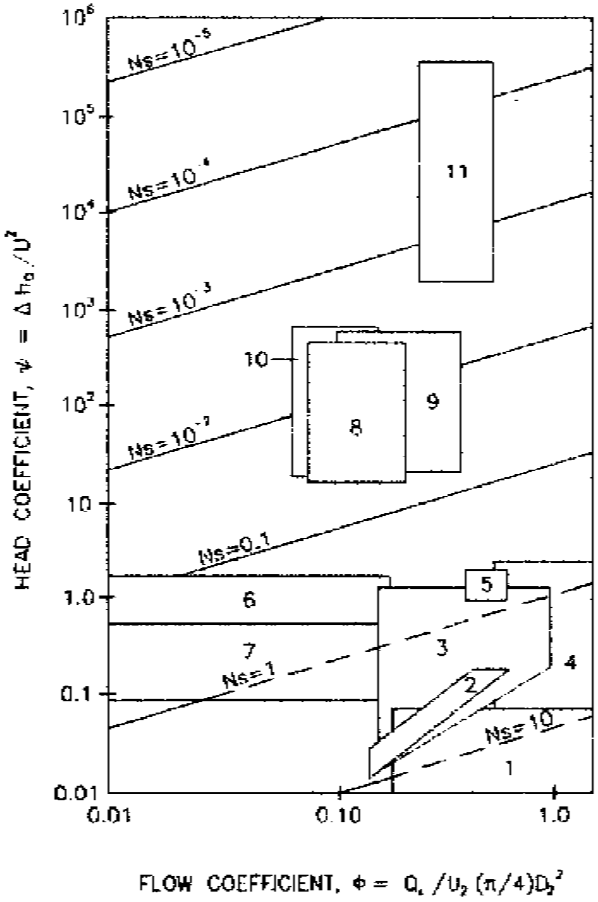
Steep, flat or unstable characteristic curve

Pump characteristics

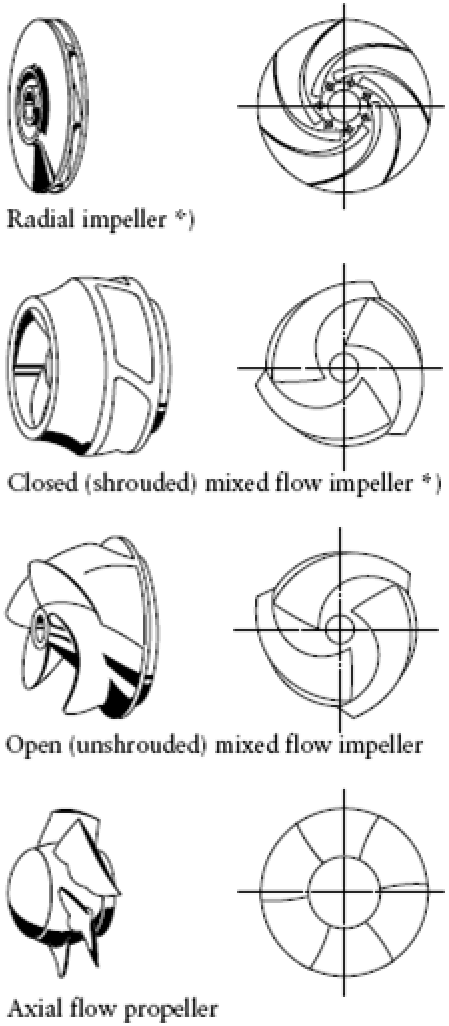
- *H*/*Q* characteristics normally stable (the developed head falls as the flow rate *Q* in-creases)
- For low specific speeds, the head *H* may – in the low flow range – drop as the flow rate decreases(shown by the dash line)
- This may cause problems in some applications

Specific speed (from Japixe-Baines)





Centrifugal Pumps, Fans and Compressors



Types:

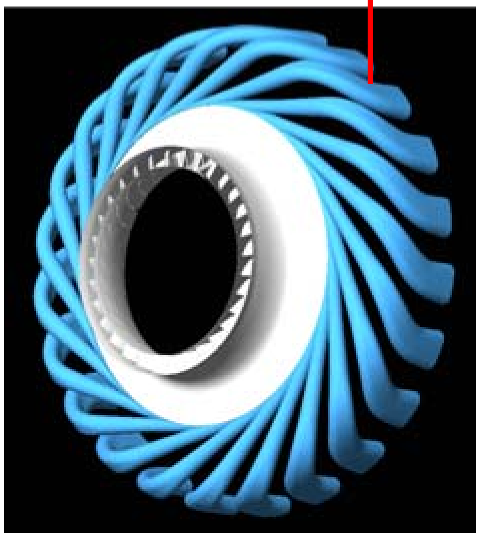
- Axial, mixed and radial flow direction
- Shrouded or unshrouded

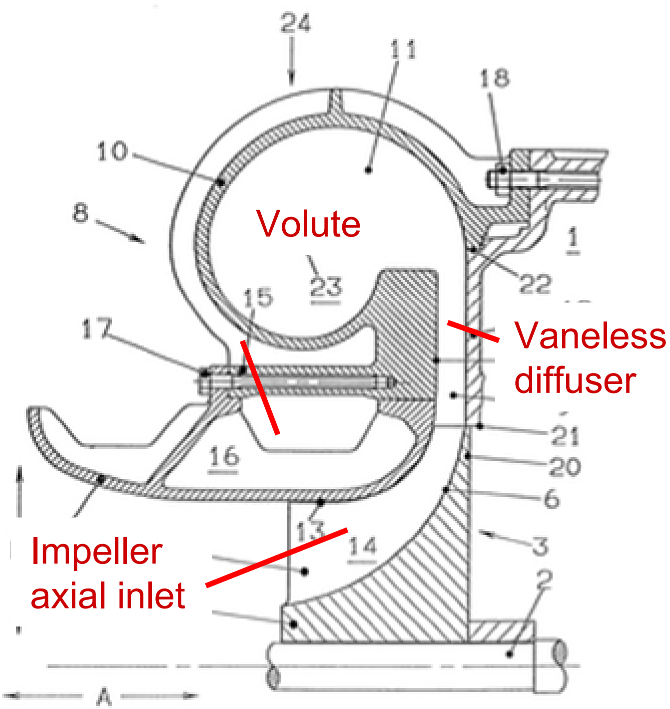
Shrouded impellers

Main components

- Impeller:
  - 2D, 3D, Backsweep?
  - Axial inlet, radial inlet
- Diffuser:
  - Vaneless, Vaned (Vane type)?
  - Diffuser ratio

Pipe diffuser





Airfoil diffuser

Centrifugal Pumps, Fans and Compressors

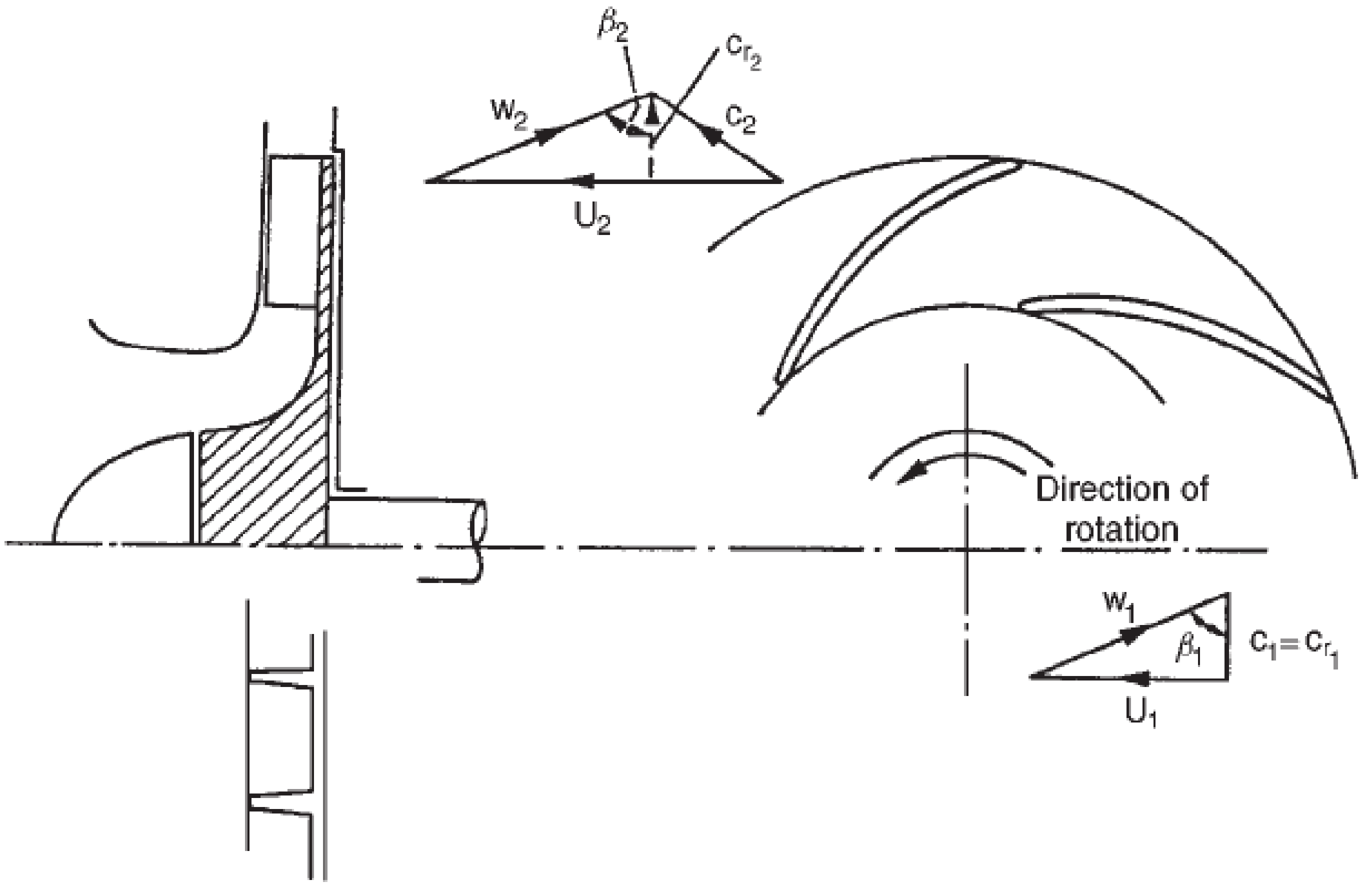


FIG. 7.2. Radial-flow pump and velocity triangles.

Centrifugal Pumps, Fans and Compressors

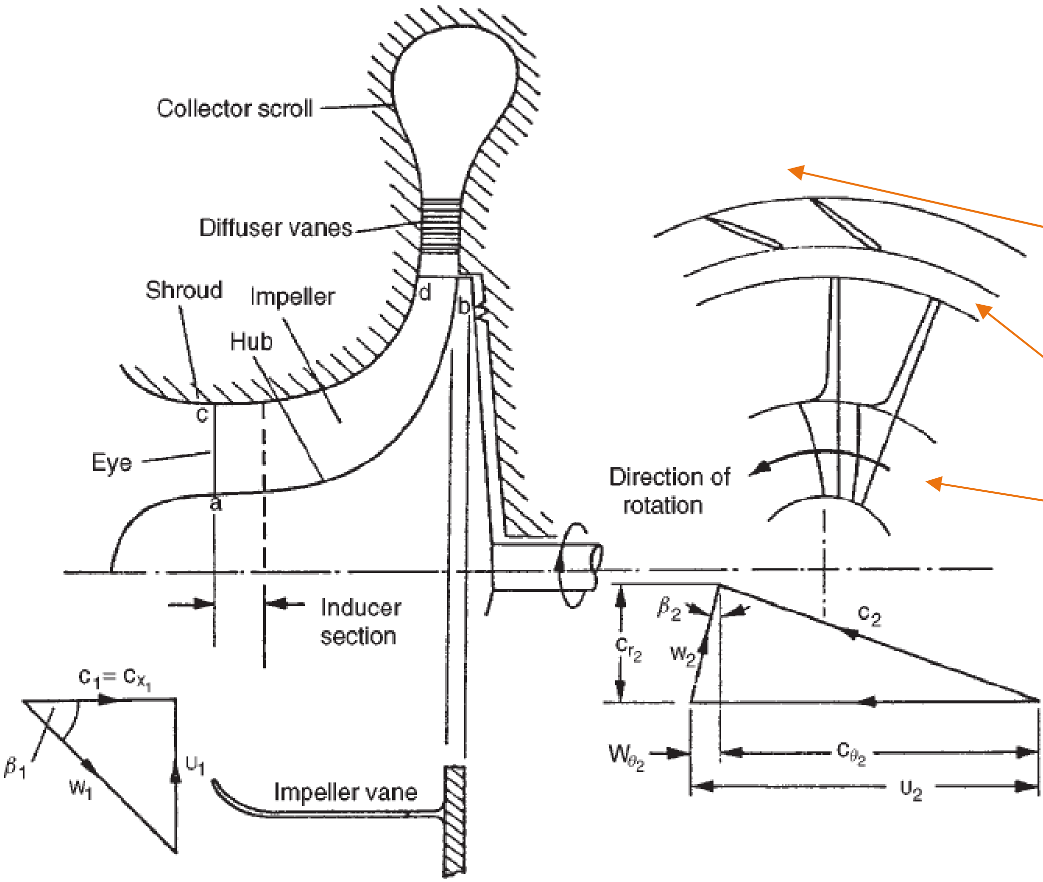
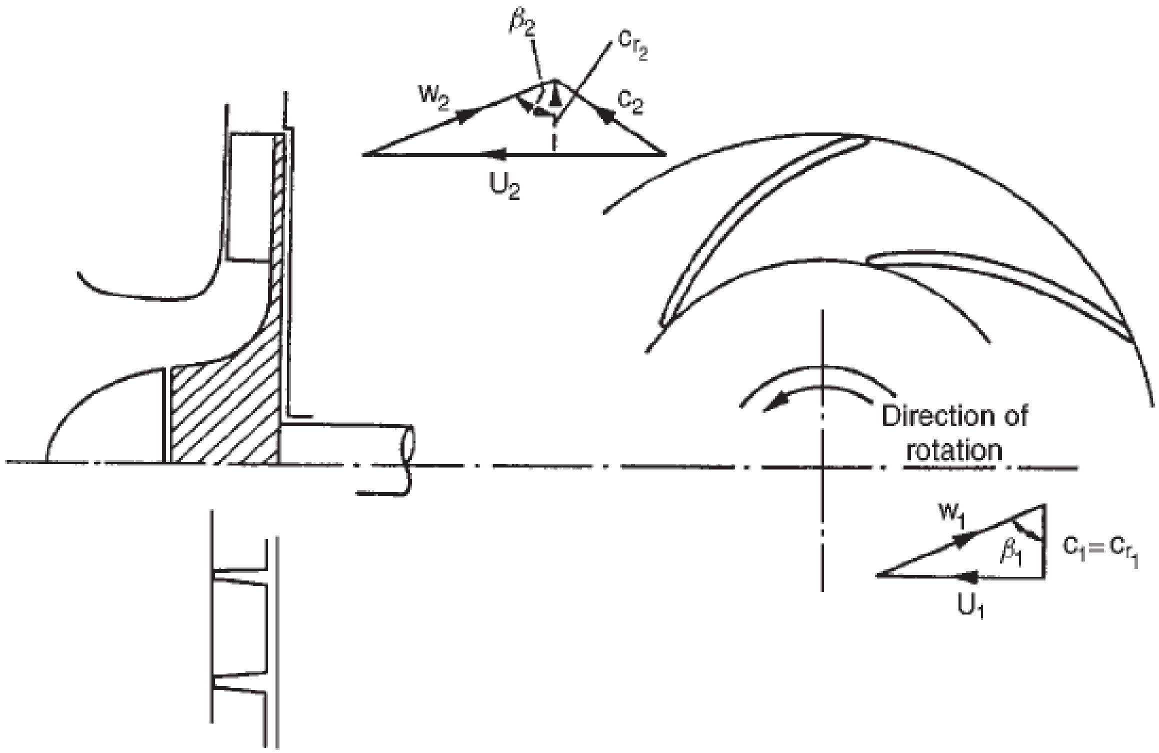


FIG. 7.1. Centrifugal compressor stage and velocity diagrams at impeller entry and exit.

Centrifugal Pumps, Fans and Compressors



$$W_x = gH = U_2 \cdot c_{\theta 2} - U_1 \cdot c_{\theta 1} = \left[c_{\theta 1} = 0 \right] = \underline{U_2 \cdot c_{\theta 2}}$$

Centrifugal Pumps, Fans and Compressors

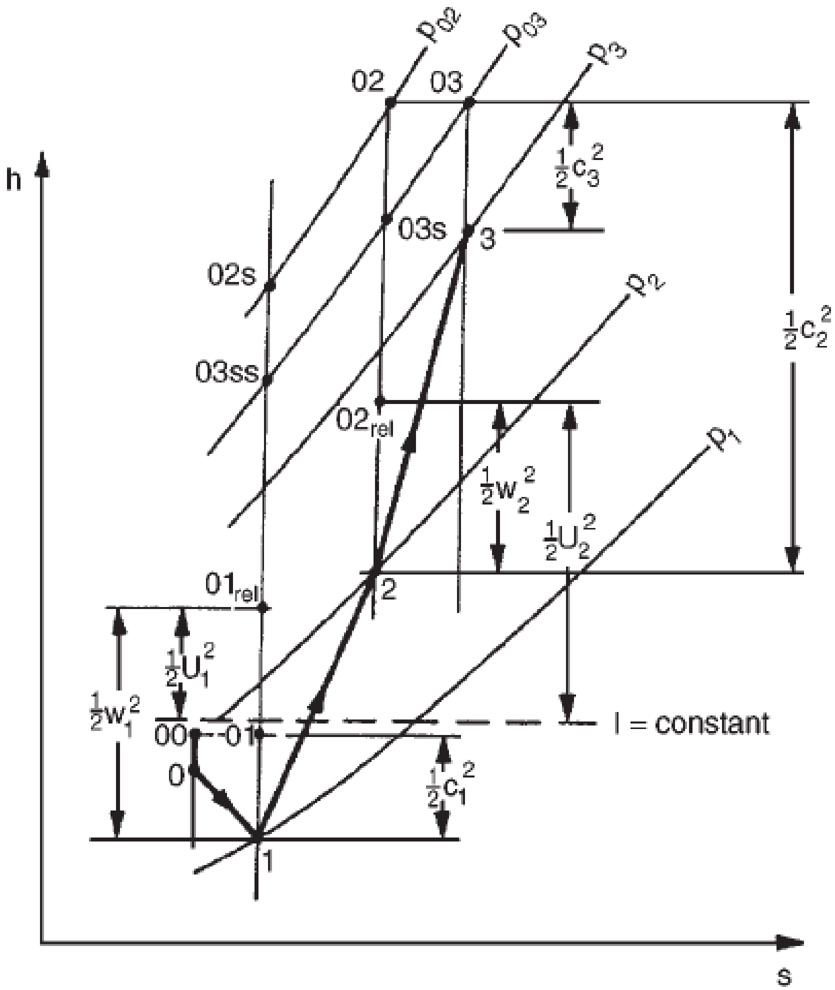
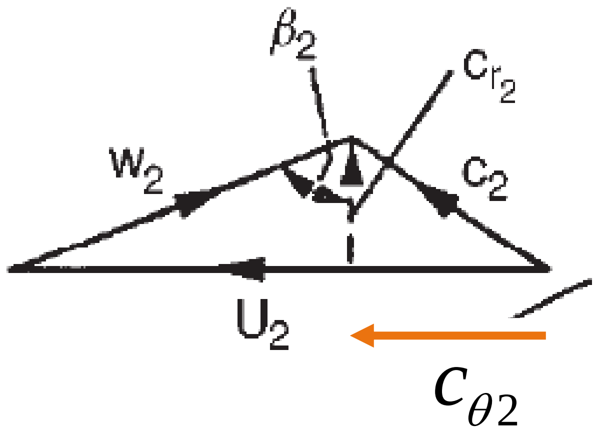


FIG. 7.3. Mollier diagram for the complete centrifugal compressor stage.

Centrifugal Pumps, Fans and Compressors

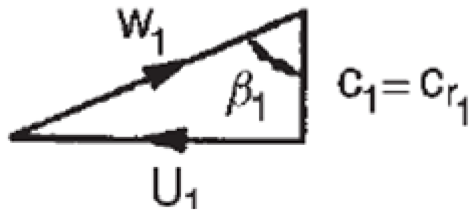
$$c_{r2}^2 = c_2^2 - c_{\theta 2}^2 \qquad \textit{Right triangle}$$

$$c_{r2}^2 = w_2^2 - \left(U_2 - c_{\theta 2}\right)^2 \qquad \textit{Left triangle}$$



Setting them equal:

$$c_2^2 - c_{\theta 2}^2 = w_2^2 - \left(U_2^2 - 2U_2 c_{\theta 2} + c_{\theta 2}^2\right)$$



Solving for $U_2 c_{\theta 2}$

$$\boxed{U_2 c_{\theta 2} = \frac{c_2^2 + U_2^2 - w_2^2}{2}}$$

$\boxed{U_1 c_{\theta 1} \text{ the same way}}$

Centrifugal Pumps, Fans and Compressors

$$W_x = gH = U_2 \cdot c_{\theta 2} - U_1 \cdot c_{\theta 1} = \frac{1}{2}\left[\underbrace{\left(c_2^2 - c_1^2\right)}_{\text{Change in dynamic pressure}} + \underbrace{\left(U_2^2 - U_1^2\right) - \left(w_2^2 - w_1^2\right)}_{\text{Change in static pressure}}\right]$$

Reaction:

$$R = \frac{\text{change in static pressure}}{\text{total pressure change}} = \frac{\left(U_2^2 - U_1^2\right) - \left(w_2^2 - w_1^2\right)}{\left(c_2^2 - c_1^2\right) + \left(U_2^2 - U_1^2\right) - \left(w_2^2 - w_1^2\right)}$$

Example

Compare 2 pumps at

1. Same inlet velocity, radially directed: $c_{\theta 1} = 0,\ \ c_{r1} = c_1$
2. Constant radial velocity: $c_{r1} = c_{r2}$
3. Same speed of rotation and same inner and outer diameter

Consequences

1. Work: $gH = U_2 \cdot c_{\theta 2} - U_1 \cdot c_{\theta 1} = \left[c_{\theta 1} = 0\right] = U_2 \cdot c_{\theta 2}$
2. Change in dynamic pressure:

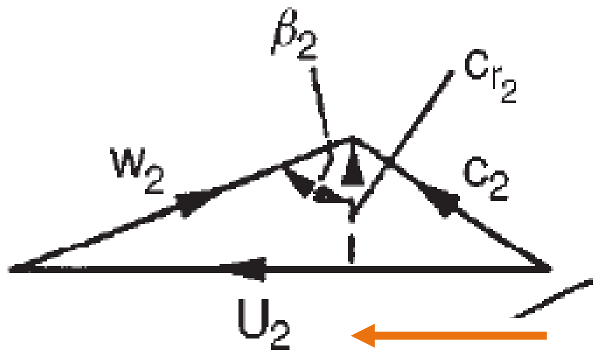
$$\Delta P_d = \frac{\rho}{2}\left(c_2^2 - c_1^2\right) = \frac{\rho}{2}\left(c_{r2}^2 + c_{\theta 2}^2 - c_1^2\right) = \frac{\rho c_{\theta 2}^2}{2}$$

Example

The total pressure rise $\Delta P_t = \rho g H = \rho \cdot U_2 \cdot c_{\theta 2}$

The ratio of the dynamic and total pressure drop becomes:

$$\frac{\Delta P_d}{\Delta P_t} = \frac{\rho c_{\theta 2}^2}{2\rho U_2 \cdot c_{\theta 2}} = \frac{c_{\theta 2}}{2U_2}$$

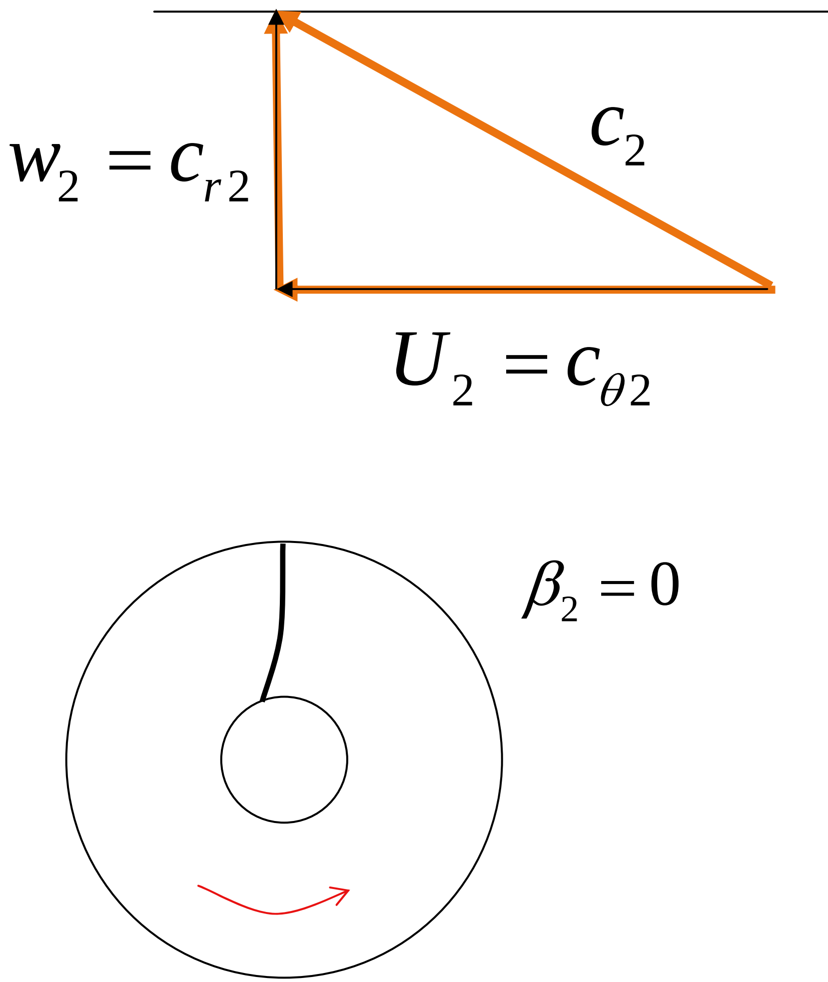


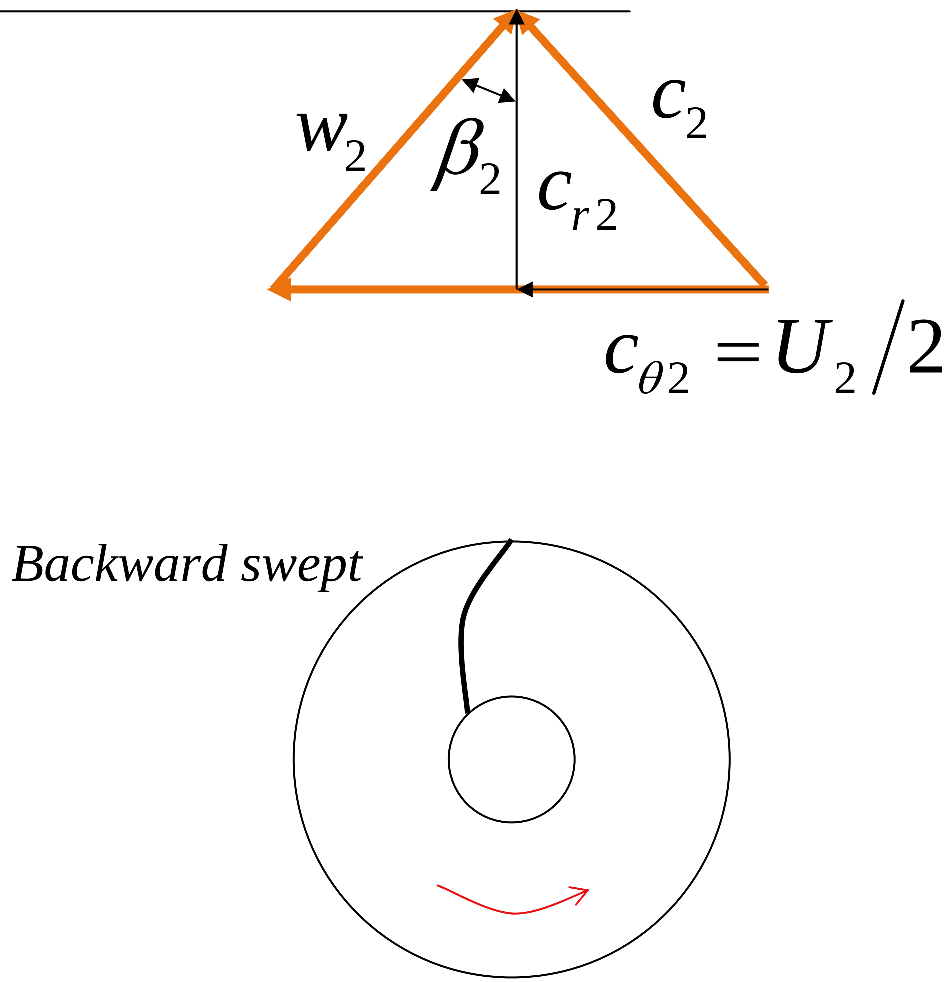
$c_{\theta 2}$

At a fixed c_{r2} :

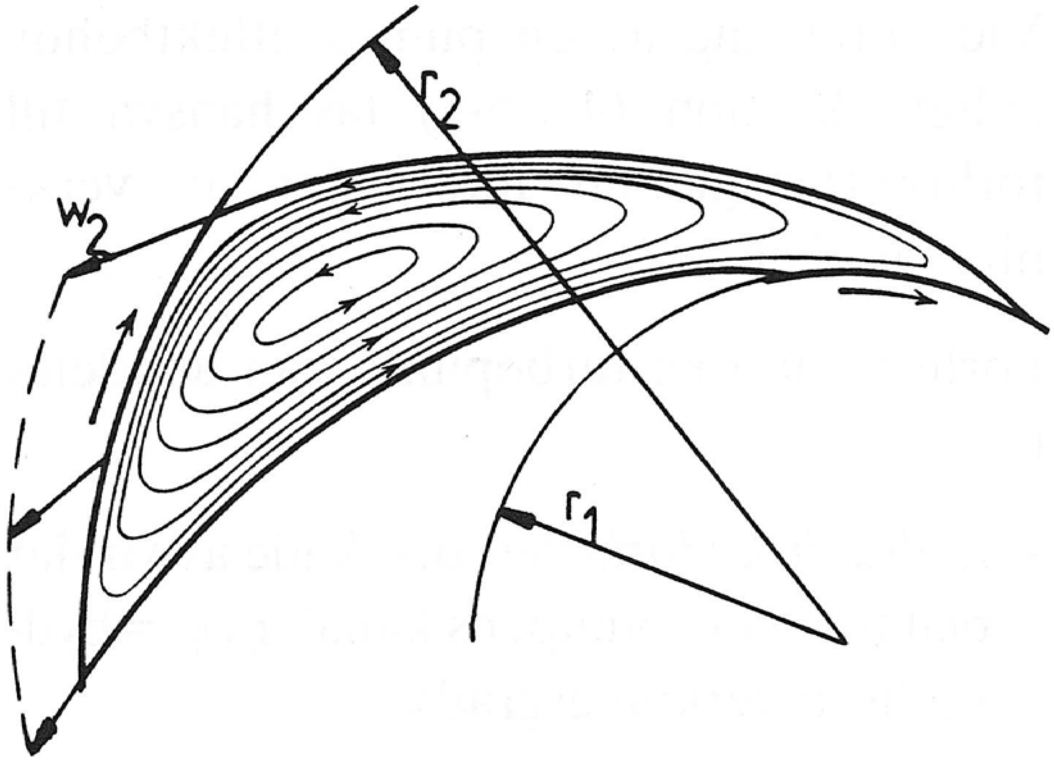
large β will decrease the dynamic part of the pressure rise

Example


$w_2 = c_{r2}$
c_2
$U_2 = c_{\theta 2}$
$\beta_2 = 0$

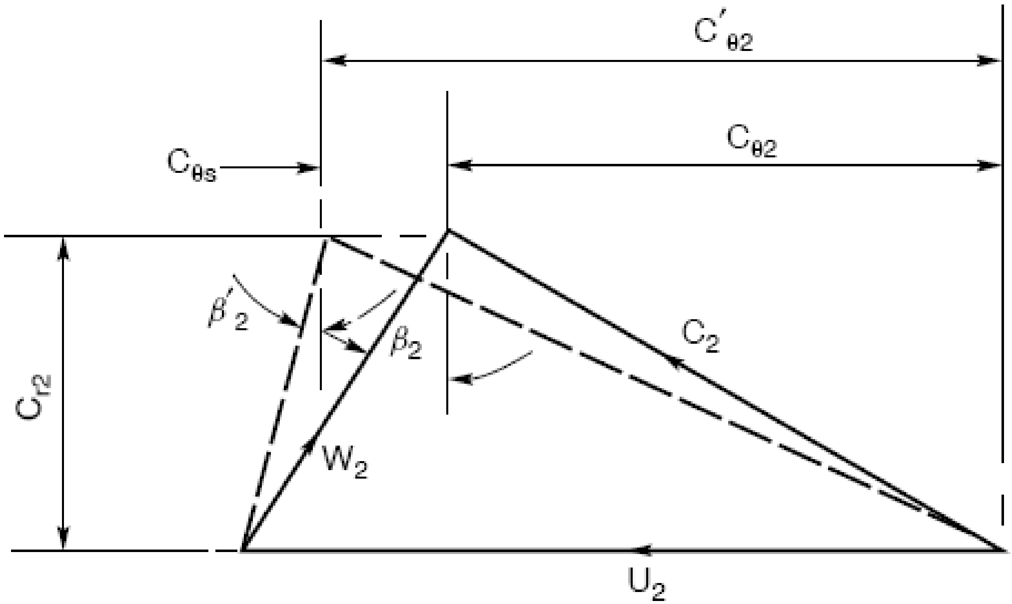

w_2
β_2
c_2
c_{r2}
$c_{\theta 2} = U_2/2$
Backward swept

Slip



$c_{\theta 2}$ reduced $\Rightarrow gH$ decreases

Centrifugal Pumps, Fans and Compressors



β_2' is the vane angle

β_2 is the average relative flow angle

FIG. 7.7. Actual and hypothetical velocity diagrams at exit from an impeller with back swept vanes.

Centrifugal Pumps, Fans and Compressors

Stodola: A relative eddy with angular velocity $\Omega = U_2 / r_2$

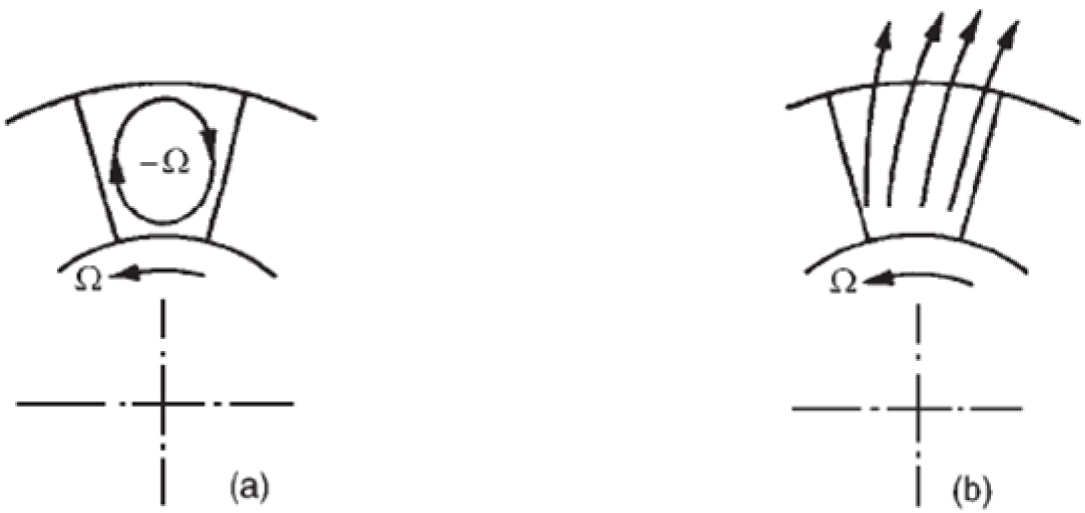


FIG. 7.8. (a) Relative eddy <u>without</u> any throughflow. (b) Relative flow at impeller exit (throughflow added to relative eddy).

Centrifugal Pumps, Fans and Compressors

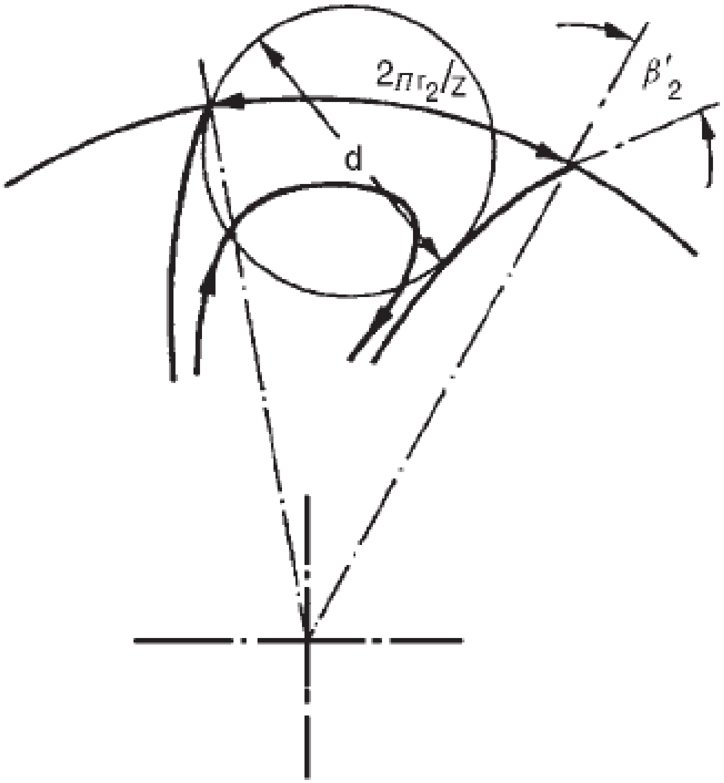


FIG. 7.9. Flow model for Stodola slip factor.

- Slip velocity is the product of the relative eddy and the radius of a circle which can be inscribed within the channel

$$c_{\theta s} = c'_{\theta 2} - c_{\theta 2} = \Omega d/2$$

- With Z being the number of vanes:

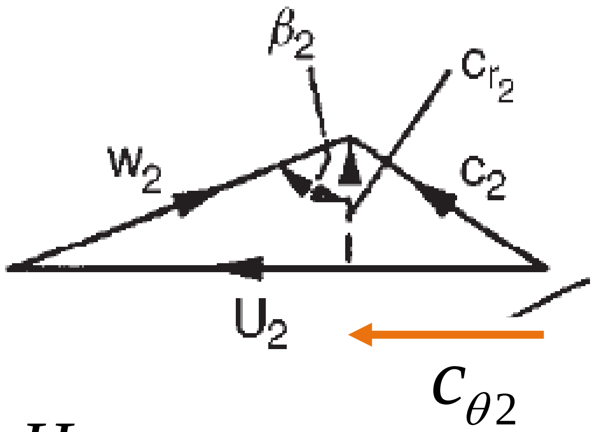
$$d \simeq (2\pi r_2/Z)\cos\beta'_2$$

Example (same assumptions as before)

Head- Volume characteristics

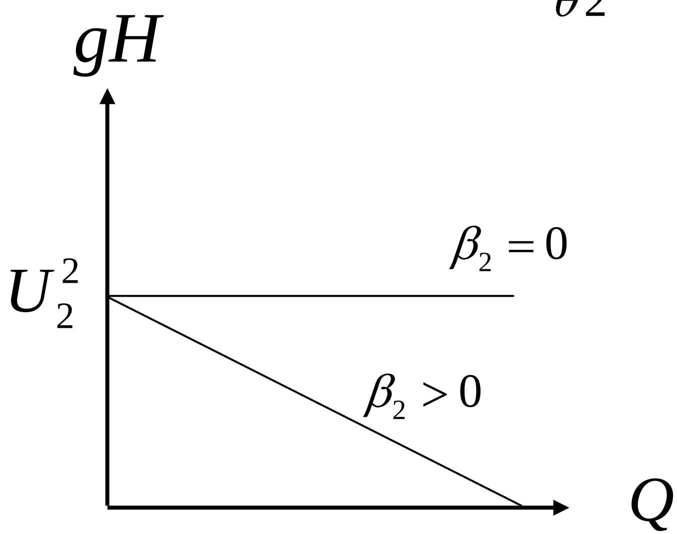
$$Q = A_2 \cdot c_{r2}$$

where A_2 is the exit area of the impeller



$$gH = U_2 \cdot c_{\theta 2}$$

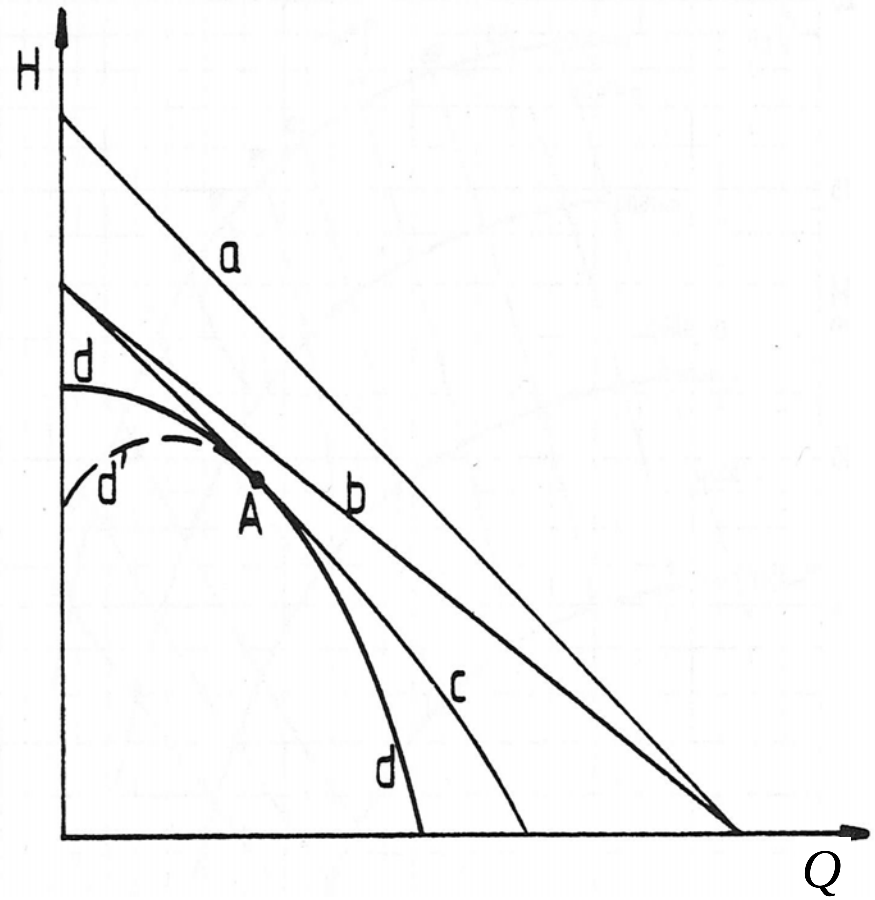
$$c_{\theta 2} = U_2 - c_{r2} \tan\beta_2 = U_2 - Q\tan\beta_2 / A_2$$



Combining these equations:

$$gH = U_2 \cdot \left(U_2 - Q\tan\beta_2 / A_2\right)$$

Example



Losses:

a-b: Slip (finite number of vanes)

b-c: Friction $\propto Q^2$

c-d: Other losses from 3d flows

d´ Instable region

Operating point

Pipe flow friction factor

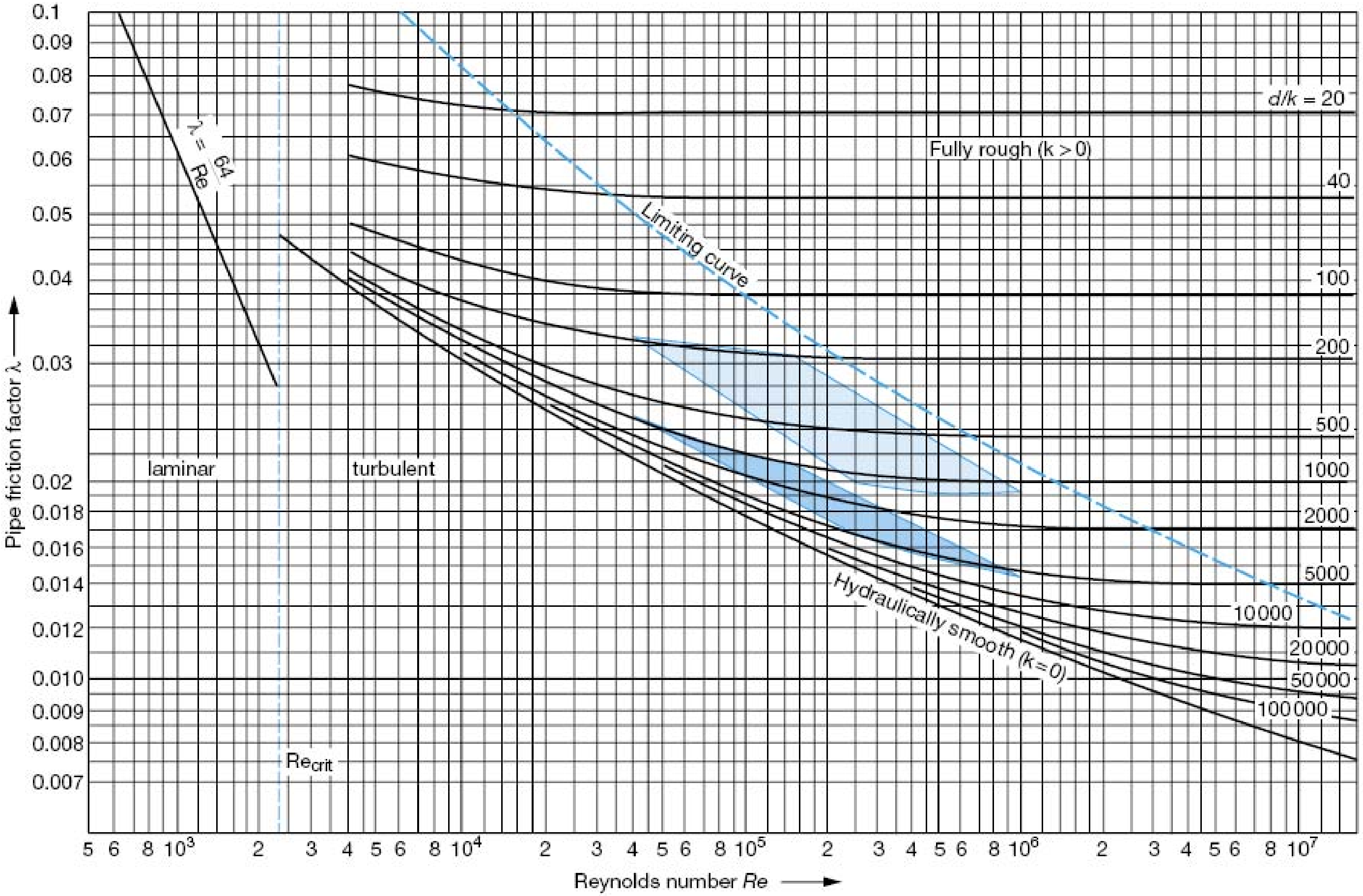


Table 3: Approximate average roughness height k (absolute roughness) for pipes