

Immersive Linear Algebra

– The world’s first linear algebra book with interactive figures

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ABSTRACT: This paper describes the development of the world’s first linear algebra book with interactive figures.

1 INTRODUCTION

At university level, the mathematics education starts with courses in analysis and linear algebra. Whereas analysis is a topic that is familiar for the students, the topics in linear algebra are mostly new. These topics include solving systems of linear equations, theory of linear vector spaces, matrix theory and theory of determinants. Already the Babylonians knew how to solve a system of two linear equations in two unknowns. In the 2000-year-old Chinese textbook “Nine Chapters of the Mathematical Art”, chapters 7 and 8 treat methods for solving systems of linear equations. Problem 7 in chapter 8 reads:

“Now there are 5 cattle and 2 sheep costing 10 liang of silver, 2 cattle and 5 sheep costs 8 liang of silver. Tell: What is the cost of a cow and a sheep, respectively?”

The theory of determinants was developed during the 17th century and onwards by Seki Takakazu in Japan, by Gottfried Leibnitz in Germany and others. The concepts of matrices were developed later. During the first half of the 20th century several modern textbooks were written. Today an introductory course in linear algebra describes and connects all of the four topics above. Considering the long history of books in linear algebra, writing a new textbook on the topic is thus somewhat daunting.

Many of the new concepts in linear algebra are geometrical in nature. One needs to understand and visualize things like points, lines, planes, projections and vectors both 2D and 3D. In writing this immersive linear algebra book our idea has been to develop interactive figures so as to ease the understanding of the new concepts.

2 DEVELOPMENT

The interactive linear algebra book (immersivemath.com) was developed over the span of three years by a three-man team with academic backgrounds in mathematics, computer graphics, computer vision and image coding. The authors have worked on the project mostly on a sporadic basis, though one of us has spent a 6-month period focusing the majority of his time on the book and our tools. In the project several tools were used to render the text, the mathematical equations and the interactive figures. The bulk of the book is written in LaTeX-style format. This allows for writing of the text and the mathematical equations in a way that is similar to what a traditional textbook would be written. A custom-made program was developed to parse our augmented LaTeX source to HTML5. An open source library, MathJax, was used to display the mathematical equations. The figures were programmed using JavaScript using a graphics engine that we have developed for the interactive illustrations.

3 RELEASE AND RECEPTION

The first four chapters of the book were released in the late evening of September 7, 2015. Within the first 24 hours the book had generated 20 000 views. The news of the book travelled fast and ended up on forums such as Hacker News. Since this initial onrush there is a steady stream of users to the site, with occasional peaks for example when new material is released. We have also been interviewed by the local news station, Sydnytt, which featured a 3-minute clip on the 2015-10-26. To date, we have had over 140,000 page views from over 45,000 users. We have also seen that the University of Hawaii is referring to our book in their education. The fifth chapter was released in October and we are in the process of releasing chapter six.

4 USE IN TEACHING

Since the book is not finished, we do not expect the book to be used in teaching yet, although the material can be used partly and the interactive figures can be used e g during lectures. We are now eager to finish the book and to see how it can be used in teaching. One interesting question is to try to explore, investigate and measure to what extent the interactivity of the figures can be used to enhance the learning process. These figures probably need to be improved and modified to optimize the understanding.

5 SUMMARY AND ACKNOWLEDGMENTS

The paper describes our efforts to develop an on-line and interactive textbook for linear algebra. In the process we have developed tools that can be used for writing such books and for simplifying the development of such interactive figures.

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