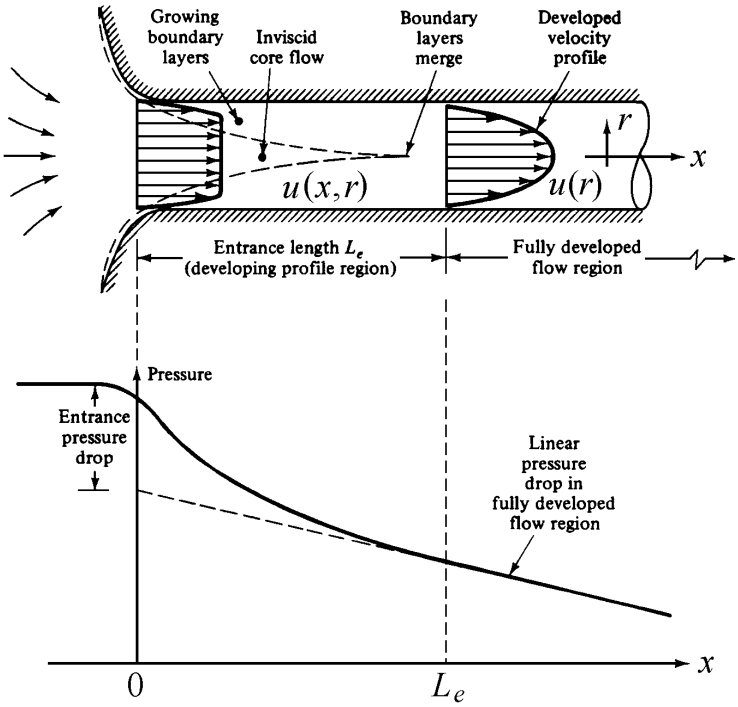
MMV211 Fluid Mechanics	LABORATION 2a	Pipe Flow Systems

MOTIVATION

Viscous flow in pipes or ducts appears in many technical applications, e.g., district heating systems, pipelines, cooling systems, ventilation ducts and power plants. When designing such a system it is great importance to determine or estimate the pressure losses that occur due to wall friction and other irreversible flow processes, e.g., for selecting a proper fan or pump to maintain the flow, a flow that is often turbulent and at high Reynolds number, which means that experimentation is crucial for this area of fluid mechanics.



PURPOSE

The purpose of this laboratory exercise is to determine and analyse

- Volume flow rate
- Pressure losses in a simple pipe system
- Pressure loss coefficient of different pipe components
- Friction factor of pipes with smooth and rough surfaces
- Velocity profile

The measurements are carried out using air as the flowing medium.

PREPARATION

For preparation; read this PM and sections 6.1, 6.3, 6.6 & 6.9 in the textbook (F. M. White, Fluid Mechanics). Specific page references relate to the 6th edition.

C. Norberg, 16/01/2010

1. THEORETICAL

1.1 Velocity profiles

Assume a fluid that flows through a pipe of constant cross-sectional area *A*. Further assume that the flow is fully developed, stationary and incompressible, $\rho = const$. Stationary conditions means that the mass flow rate $\dot{m}$ is constant. Since the fluid density ρ is constant that goes also for the volume flow rate, $Q = \dot{m}/\rho = VA = const.$, where V is the mean velocity. The velocity profile or velocity distribution depends on the prevailing Reynolds number,

$$\mathrm{Re} = \frac{\rho V D_h}{\mu} = \frac{V D_h}{\nu} \tag{1}$$

D_h is the hydraulic diameter,

$$D_h = \frac{4A}{\mathcal{P}} \tag{2}$$

where $\mathcal{P}$ is the periphery, the "wetted" circumference. Unless noted otherwise, from here on the cross-section of the pipe is assumed circular, $D_h = d = 2R$.

At "normal" technical conditions the flow can be considered to be laminar if Re ≤ 2100 and fully turbulent if Re > 4000, approximately. The figure below shows a schematic representation of turbulent and laminar velocity distributions at approximately equal flow rates.

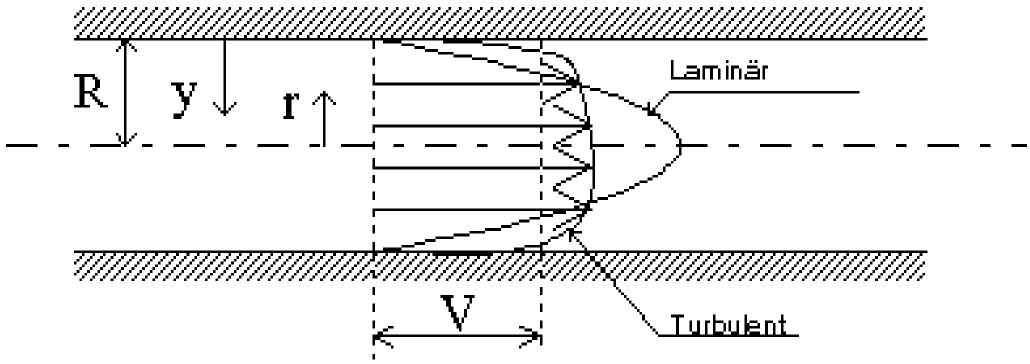


Figure 1: Laminar and turbulent pipe flow.

In the transitional regime, i.e., Reynolds numbers between 2100 and 4000 (approx.), the local flow is intermittent, laminar at times and turbulent at other times. However, at technical/engineering conditions, the changeover from laminar to fully turbulent flow can be considered to occur abruptly at a critical Reynolds number of about 2300. When designing pipe systems the regime in between Re = 2100 and 4000 (approx.) should be avoided.

1.2 Mean velocity and volume flow rate

The local (time-mean) velocity *u* can be determined by using a Pitot tube in combination with a static pressure wall tap, see figure 3. When the Pitot tube is pointed towards the oncoming flow the fluid is decelerated to zero velocity, the static pressure of the fluid increases to the stagnation pressure, p_0. The deceleration is assumed to be frictionless. Along the assumed horizontal stagnation streamline of the Pitot tube, the Bernoulli equation gives

$$p_0 = p + \rho \frac{u^2}{2} \tag{3}$$

Hence, the local velocity at radius *r* is simply

$$u(r) = \sqrt{\frac{2(p_0 - p)}{\rho}} \tag{4}$$

where *p* is the static pressure along the streamline in the undisturbed flow. Since the (time-mean) flow is assumed to be rectilinear this static pressure is equal to the static pressure at the pipe wall at the same horizontal heigth as the tip of the Pitot tube, again neglecting viscous effects. (In air measurements the effects of gravity can be neglected so this positioning of the wall tap is not crucial.)

Viscous effects can be neglected (error less than 0.5%) when the Reynolds number, based on the outer diameter of the Pitot tube, is greater than about 100. For air at atmospheric conditions and a Pitot tube diameter of 1.5 mm this means a lower limit of about 1 m/s.

At stated conditions, the mean velocity V can be determined from the velocity profile,

$$Q = \pi R^2 V = 2\pi \int_0^R u(r)\, r\, dr \Rightarrow V = \frac{2}{R^2} \int_0^R u(r)\, r\, dr \tag{5}$$

Dividing with the velocity at the centre u_0 $(= u_{max})$ and introducing $\eta = r / R$ gives

$$\frac{V}{u_0} = 2\int_0^1 \frac{u}{u_0} \eta \, \mathrm{d}\eta \tag{6}$$

For fully developed laminar flow the velocity profile is parabolic:

$$\frac{u}{u_0} = \left[1 - \left(\frac{r}{R}\right)^2\right] = 1 - \eta^2 \tag{7}$$

This gives

$$\left(\frac{V}{u_0}\right)_{\mathrm{lam}} = 0.5 \tag{8}$$

In fully developed turbulent flow, see p. 151 in White, the time mean velocity varies approximately according to the expression below, except close to the wall:

$$\frac{u}{u_0} \approx \left(1-\frac{r}{R}\right)^m = (1-\eta)^m \qquad (9)$$

where $0.1 < m \leq 0.2$ (m decreases with increasing Reynolds number). This gives

$$\left(\frac{V}{u_0}\right)_{\text{turb}} \approx \frac{2}{(1+m)(2+m)} = 0.76-0.87 \qquad (10)$$

Alternatively, the integral in eq. (6) can be calculated using numerical or graphical methods. This requires several measurement values across the diameter of the pipe. To find the mean velocity, a quick but often uncertain method is to use the experimental fact that the local velocity at about a quarter radius from the wall ($y/R = 0.24$) is equal to the mean velocity, see figure 2. The potential uncertainty comes about from the difficulty in finding this position with high accuracy, which is needed because of the velocity variations at around this point.

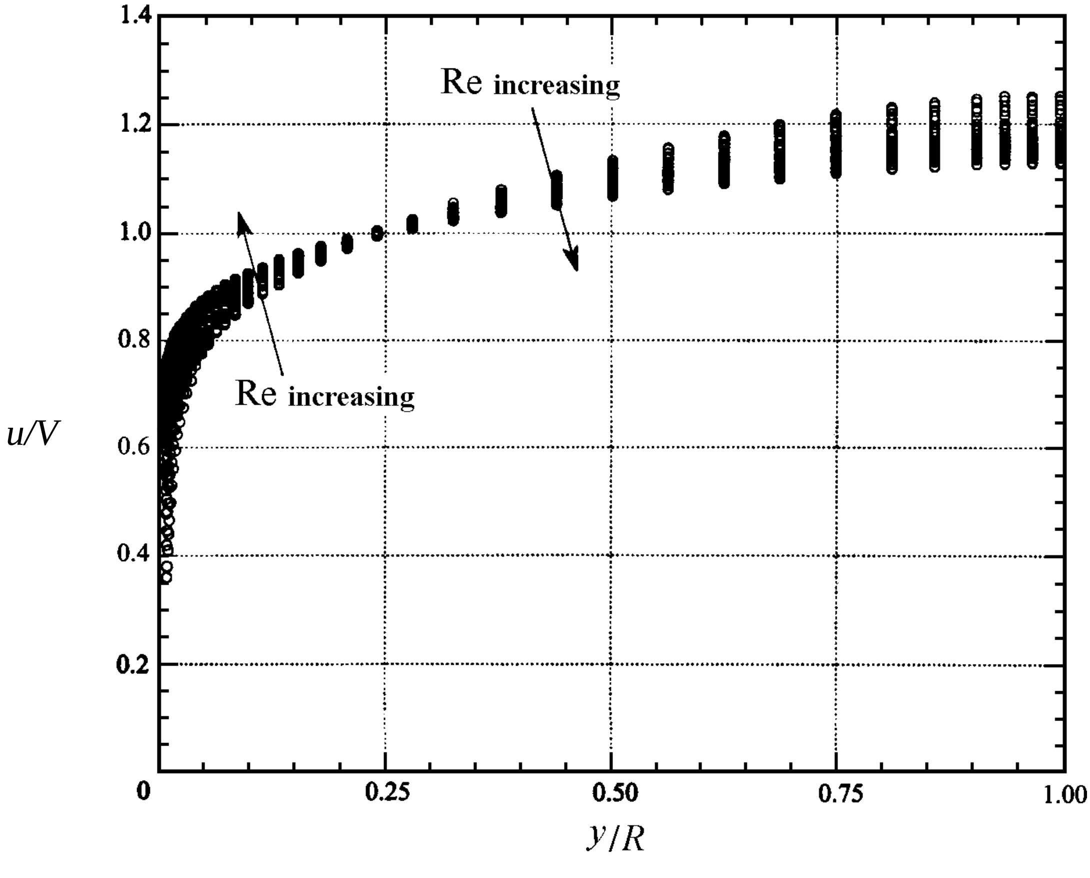


Figure 2: Velocity profiles from water measurements in a smooth pipe (d = 2R = 129 mm, 26 profiles, Re = 31x10^3 – 35x10^6). The local (time-averaged) velocity at y/R = 0.24 is approximately equal to the mean velocity; from M. V. Zagarola & A. J. Smits, Mean-flow scaling of turbulent pipe flows. *Journal of Fluid Mechanics* **373**, 33-80, 1998.

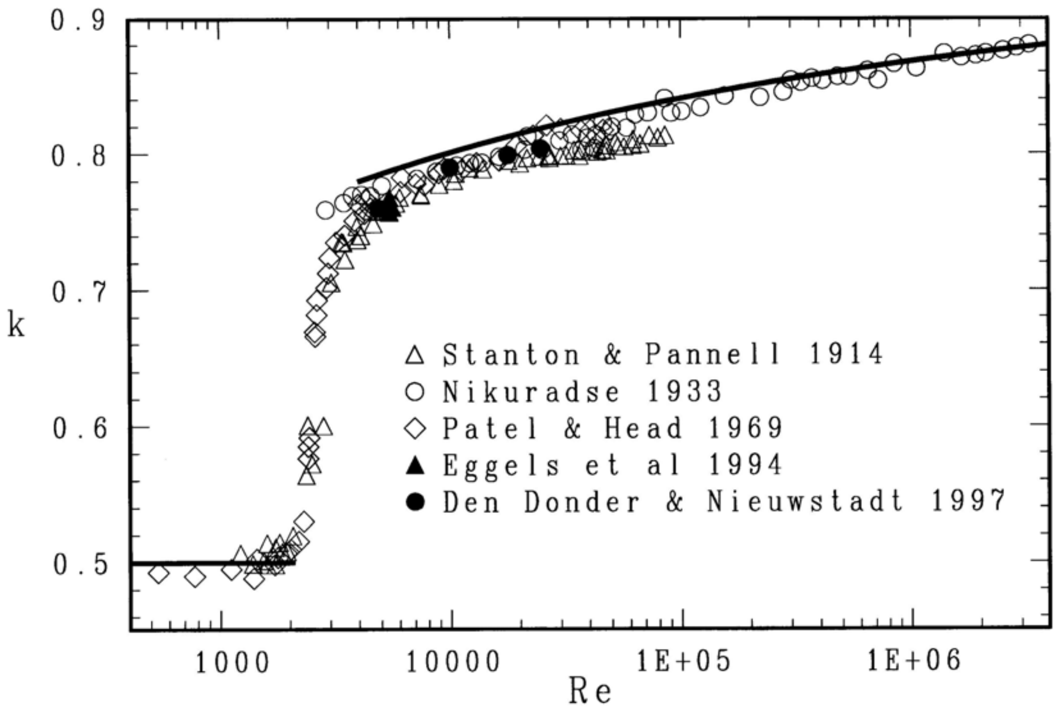


Figure 3: Ratio between mean and centre velocity for fully developed flow in smooth pipes, $k = V / u_0$.

Another alternative is to make use of the diagram shown in Fig. 3, with measured ratios V / u_0 *vs.* Re (fully developed flow in a smooth pipe). From a measurement of u_0, the mean velocity V can be found by iteration.

The mean velocity can also be determined using some type of flow meter. During the lab exercise the volume flow rate will be measured using the following methods,

1. Rotameter
2. Thin-plate orifice
3. Integration of the velocity profile

1.3 Pressure losses

The extended Bernoulli equation (the energy equation) between two arbitrary pipe sections 1 and 2, assuming incompressible stationary flow reads

$$p_1 + \alpha_1 \frac{\rho V_1^2}{2} + \rho g z_1 = p_2 + \alpha_2 \frac{\rho V_2^2}{2} + \rho g z_2 + \Delta p_\mathrm{f} + \rho w_s \qquad (11)$$

where p and z denote sectional mean values of static pressure and the vertical height above some reference level; Δp_f is the (total) irreversible pressure loss; w_s the technical work per unit mass (negative for pumps and fans and positive for turbines) and α a kinetic energy correction factor that depends on shape of the velocity profile (fully developed laminar flow, $\alpha = 2$, see White, p. 180). The pressure loss is divided into two groups: (i) frictional losses, which are solely due to wall friction (major losses), and (ii) losses due to formation of vortices and turbulence in valves and sudden expansions, streamline curvature effects in elbows and bends, etc. (minor losses).

1.3.1 Major losses; friction factor

Over a certain pipe length *L*, the pressure loss due to friction, the so-called major loss, at fully developed flow conditions, is given by the Darcy-Weisbach equation:

$$\Delta p_{\mathrm{f}} = f \frac{L}{d} \frac{\rho V^2}{2} \tag{12}$$

where f is the friction factor, which is a function of the Reynolds number and the relative roughness, ε / d. (ε is the equivalent roughness height that is determined empirically, usually provided by the pipe manufacturer; it may also be found in handbooks, see Table 6.1 in White). In fully developed laminar pipe flow ($\mathrm{Re} \le 2100$) the friction factor equals

$$f = \frac{64}{\mathrm{Re}} \tag{13}$$

Thus, the laminar friction pressure loss is independent of the fluid density and surface roughness (within reasonable limits), and proportional to the dynamic viscosity and the flow rate. In fully developed turbulent pipe flow (approx. $\mathrm{Re} > 4000$) and for engineering purposes, the formula of Haaland (1983) can be used for calculation of the friction factor. The formula, eq. (6.49) in White, reads

$$\frac{1}{f^{1/2}} = -1.8 \log\left[\frac{6.9}{\mathrm{Re}} + \left(\frac{\varepsilon / d}{3.7}\right)^{1.11}\right] \tag{14}$$

1.3.2 Minor losses; loss coefficient

The pressure loss due to a pipe component, a so-called minor loss, is expressed as:

$$\Delta p_{\mathrm{f}} = K \frac{\rho V^2}{2} \tag{15}$$

where *K* is the loss coefficient, usually based on the mean velocity upstream. In most turbulent flow, the loss coefficient can be regarded as a constant.

1.3.3 Total pressure loss

The total pressure loss is the sum of frictional losses (major losses) and those due to pipe components (minor losses),

$$\Delta p_{\mathrm{f}} = \frac{\rho}{2}\left[\sum_i \left(f \frac{L}{D_h} V^2\right)_i + \sum_j \left(K V^2\right)_j\right] \tag{16}$$

where the summations are for different parts along the pipe system.

1.3.4 Fluid properties

Air at normal pressures and temperatures can be considered to be an ideal gas. The air density then is

$$\rho = \frac{p}{RT} \tag{17}$$

where $R = 287$ J/(kg K). Temperature T should be in kelvin, $T(\text{K}) = T(°\text{C}) + 273.15$.

The dynamic viscosity of gases can be calculated from Sutherland's formula,

$$\frac{\mu}{\mu_0} = \left(\frac{T}{T_0}\right)^{3/2} \left(\frac{T_0+S}{T+S}\right) \tag{18}$$

For air: S = 110.4 K, $T_0 = 273$ K, $\mu_0 = 1.71\times10^{-5}$ Pa·s. T should be in kelvin (K).

2. DESCRIPTION OF APPARATUS

2.1 The experimental rig

The rig (figure 4) is mounted on a wooden board and is made mostly of standard PVC-pipe components, of inner diameter nominally equal to 19.4 and 38.8 mm, respectively. Two pipe sections have been made artificially rough while the rest of the pipe system is as smooth as when purchased. Various components can be installed at positions A and B.

The rig is driven using air from an in-house compressed air system (pressure ~ 7 bar). To sustain a constant flow rate during the measurement period, a tank equipped with a pressure regulator is used as an accumulator between the compressed air system and the rig. The flow is regulated using a valve mounted at the inlet of the pipe.

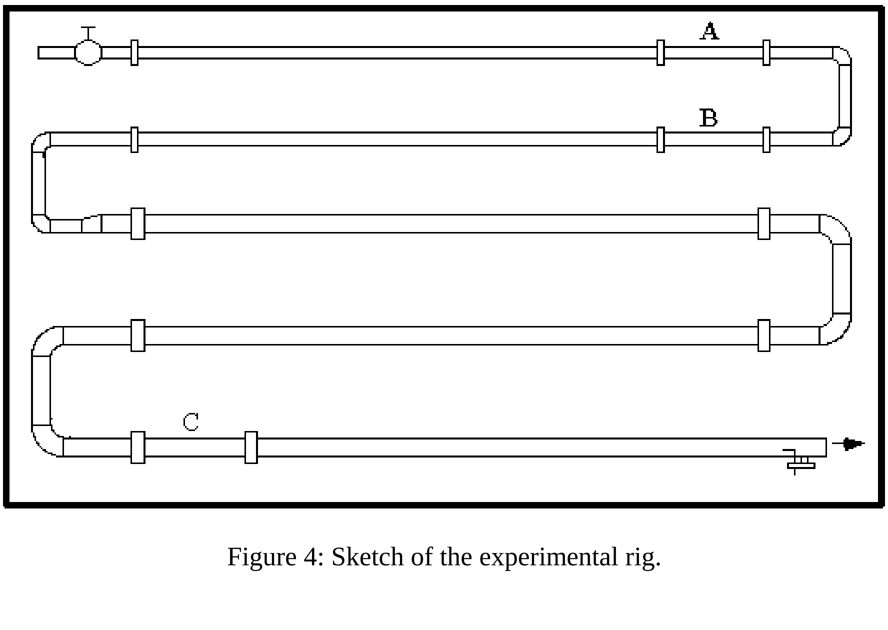


Figure 4: Sketch of the experimental rig.

The specific pipe diameters, distance between pressure measurement points and other dimensions of interests will be given at the laboratory session.

2.2 Flow measurement

The rotameter principle is the following (see White, p. 412): a specially designed pointy body is positioned in a vertically placed conical and transparent pipe. The flow entering from below lifts the body, which rotates due to its design (stabilizing the body). The conically shaped pipe keeps the distance between the body and the pipe wall constant. Hence, the height of the body becomes more or less linearly related to the flow rate. The height depends on the dimensions of the body and the pipe, the flow interval, the density and the viscosity of the fluid and also of the weight of the body, which means that it has to be calibrated. Generally this type of flow meters has an uncertainty of about 3%.

The thin-orifice plate is mounted in position C (figure 4). Upstream the orifice plate there is a honeycomb section, which decreases the effects of the pipe bends (see section 3.3). Principally the orifice plate leads to a decrease of the flow area, which increases the fluid velocity and causing a lower static pressure, see eq. (11). The pressure is also reduced due to losses, e.g., formation of swirls, which has to be compensated for when calculating the flow. The volume flow can be determined by measuring the pressure drop over the orifice plate (see White, p. 417 ff). A detailed description of the calculation procedure is given in Appendix A.

2.3 Pressure measurements

Static pressure sockets are mounted at points along the pipe system. The sockets are made of cannulas of stainless steel with an inner diameter of 1 mm. Pressure differences are measured using two different manometers.

Traversing Pitot tubes are used to measure the local velocities at the exit of the pipe. The pipe is traversed using a screw of M6×1, which means that turning the screw once corresponds to a movement of 1 mm, see figure 5. The dynamic pressure can be measured directly by connecting the Pitot tube and the static wall pressure socket to each side of the manometer ($\rho u^2 / 2 = p_0 - p$). The static pressure socket is placed approximately one half pipe diameter upstream of the Pitot tube pressure tap to minimize any disturbances of the stagnation (total) pressure close to the pipe wall.

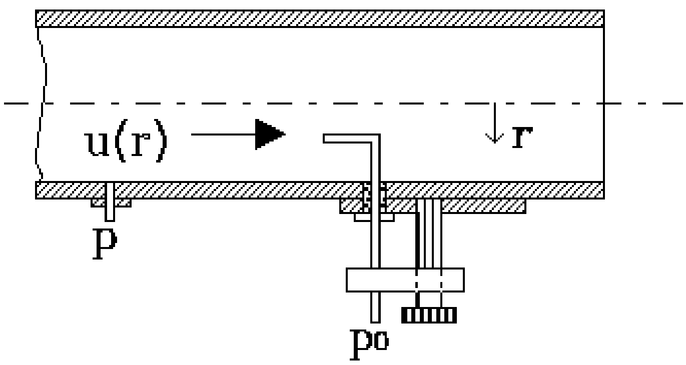


Figure 5: Traversing Pitot tube.

3. EXPERIMENTS

During the lab exercise the pressure drop for various components will be measured, at different flow rates. The friction factor and loss coefficients are then computed and the results displayed in a diagram as a function of the Reynolds number. The flow rate is monitored and measured using a rotameter. At some selected flow rates, also the methods of using a thin-plate orifice and integration of velocity profile (at the pipe exit) will be employed.

3.1 Pressure drop over pipe lengths and pipe components

Read the barometric pressure and the air temperature at the pipe system exit. Compute air density and dynamic viscosity using formulas in section 1.3.3.

The following applies for the liquid manometer (u-pipe with water):

$$\Delta p(\mathrm{Pa}) = \mathrm{reading}(\mathrm{mmH_2O}) \cdot 9.81$$

The following applies for the micromanometer (pressure ranges = 10, 100 and 1000 Pa, respectively; full scale output 5 V):

$$\Delta p(\mathrm{Pa}) = \frac{(\text{voltage reading, V})}{5\ \mathrm{V}} \cdot (\text{pressure range})$$

For each flow rate:

- Measure the pressure drop across the given components and compute the friction factor f and the minor loss coefficient K.
- Plot f and K as a function of the Reynolds number.
- Estimate the relative roughness ε / d for the examined pipes by comparing with the Moody diagram (Fig. 6.13 in White) or some equivalent formula.

3.2 Velocity profile

The velocity profile is found by traversing a Pitot tube at the pipe exit (figure 5), following a table of recommended radial positions. The pressure reading is proportional to the dynamic pressure.

Determine the non-dimensional velocity profile and sketch it.

The non-dimensional velocity, u/u_0, can be computed according to:

$$\frac{u}{u_0} = \sqrt{\frac{p_0 - p}{(p_0 - p)_{max}}} = \sqrt{\frac{(\text{pressure reading})}{(\text{pressure reading})_{max}}}$$

The flow rate, $Q = VA = V\pi d^2 / 4$, is determined by numerical integration of eq. (6).

3.3 Velocity profiles after pipe bends

(by Jonas Bolinder, previously at Division of Fluid Mechanics, LTH)

The maximum velocity is found to be shifted to the outer side of the pipe. According to the non-viscous theory the opposite behaviour could be expected, i.e. the maximum velocity is shifted to the inner side of the pipe since for a potential vortex ("line vortex" in White) it is given that $v_\theta = C / r$, where C is a constant, meaning higher velocity for smaller radius. However, for internal pipe flows in reasonable long pipes the friction has a large influence on the flow.

Briefly, the phenomena can be described accordingly: Close to the wall the fluid velocity is low relative to the velocity in the central parts of the pipe. This is due the wall friction. Hence, as the fluid passes a pipe bend the fluid in the central parts be more affected by the centrifugal force and fluid particles of a high velocity will move to the outer part of the pipe, which leads to a skewed velocity profile. At the same time a pressure difference between the outer part and the inner part of the pipe builds up with a higher pressure at the outer part. The pressure difference will drive the fluid particles back along the pipe walls to the inner part of the pipe. All in all the fluid particles will depict serpentine-shaped paths.

To simple describe the flow in a bent pipe the flow is divided into axial and secondary flow. The axial flow is the projection of the velocity vector onto the tangential of the centreline. The secondary flow is the projection onto the cross-section of the pipe. The secondary flow often consists of two symmetrical swirls, see figures below. The real velocity in each point is given as a sum of the axial and the secondary velocity, which gives the serpentine shaped (helical) paths.

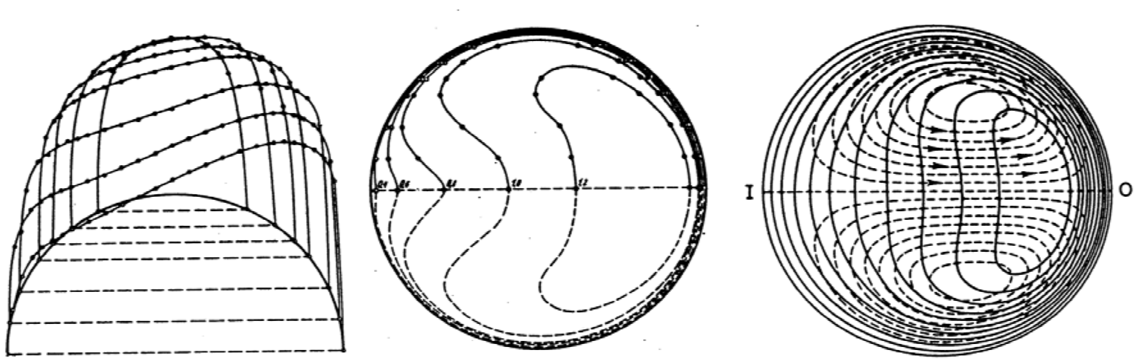


Figure 6:

Two sub-figures on left: Axial flow, $R_k/R = 50$, Re = 8540; outer part to the right (experiments of Adler, 1934).[1]	Right: Laminar axial flow (solid line) and secondary flow (dashed) (numerical calculations by McConalogue & Srivastava, 1968).[2]

[1] V. N. Adler, Strömung in gekrümmten Rohren, *Z. angew. Math. Mech.* **4**, 247-275, 1934.
[2] D. J. McConalogue & R. S. Srivastava, Flow in curved tubes, *Proc. Roy. Soc.* **307** A, 37-52, 1968.