Lecture 6. Laminar Non-premixed Flames

part 2
Content

- Structures of laminar non-premixed flames
- Mixture fraction
- Burke-Schumann flame-sheet model
- Jet diffusion flames
- Laminar diffusion flame at high mixing rate
Where is the reaction zone?

Mixing length $\ell$

Zst

Burnable mixture

Diffusion velocity: Fick’s Law

$$Y_i V_i = D_i \nabla Y_i, \quad V_i = \frac{D_i}{Y_i \nabla Y_i} \propto \frac{D_i}{\ell}$$
Where is the reaction zone?

- Post flame zone
- Preheat zone

Burnable mixture

A

O_2

Zst

F

O_2

F

F

Post flame zone

Preheat zone

Burnable mixture

A

F

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Laminar Non-premixed Flames
Where is the reaction zone?

- Preheat zone
- Post flame zone
- Burnable mixture

A \rightarrow \text{Burnable mixture} \rightarrow F
Where is the reaction zone?

- Preheat zone
- Post flame zone
- Burnable mixture

A → O₂ → F

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Where is the reaction zone?

Burnable mixture
Where is the reaction zone?

- Preheat zone
- Post flame zone

Burnable mixture

A

O₂

F

Zst
Where is the reaction zone?

- Preheat zone
- Post flame zone
- Burnable mixture
Where is the reaction zone?

- Oxidizer-rich mixing zone
- Fuel-rich mixing zone
- Reaction zone
- Burnable mixture
Where is the reaction zone?

T

T_{st}

T_{cr}

T_1

T_2

Z

0

Z_{st}

1

Fuel-lean

Fuel-rich

A

F

Burnable mixture
Burke-Schumann flame-sheet structure

Reactions occur in a zero thickness sheet at $Z=Z_{st}$

Soot problem

Reaction zone

Fuel-lean

Fuel-rich

$Z_{st}$

$Y_i$

$A$

$P$

$P$

$F$

Reactions occur in a zero thickness sheet at $Z=Z_{st}$
Burke-Schumann flame-sheet structure

Reactions occur in a zero thickness sheet at $Z = Z_{st}$.
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Laminar jet diffusion flames

A non-reactive fuel jet

Mixing layer

Air

Zst

Fuel

Zst

Burnable mixture
Laminar jet diffusion flames

\[ Y_F = \frac{Z - Z_{st}}{1 - Z_{st}}, \quad Y_A = 0, \quad Y_P = \frac{1 - Z}{1 - Z_{st}} \]

\[ Y_F = 0, \quad Y_A = 1 - \frac{Z}{Z_{st}}, \quad Y_P = \frac{Z}{Z_{st}} \]
Jet diffusion flame: mathematical description

\[
\frac{\partial \rho Y_i}{\partial t} + \nabla \cdot \rho Y_i \vec{v} = \nabla \cdot \rho D_i \nabla Y_i + \omega_i
\]

\[
\frac{\partial \rho Y_F}{\partial t} + \nabla \cdot \rho Y_F \vec{v} = \nabla \cdot \rho D_F \nabla Y_F + \omega_F
\]

\[
\frac{\partial \rho Y_P}{\partial t} + \nabla \cdot \rho Y_P \vec{v} = \nabla \cdot \rho D_P \nabla Y_P + \omega_P
\]

\[
D_F = D_P = D, \quad \omega_F + \omega_P Z_{st} = 0
\]

\[
\frac{\partial \rho Z}{\partial t} + \nabla \cdot (\rho \vec{v} Z) = \nabla \cdot (\rho D \nabla Z)
\]

\[
Z = Y_F + \frac{1}{1 + \gamma_A} Y_P = Y_F + Y_P Z_{st}
\]

Combustion

\[
Z = Y_F
\]

No combustion (pure mixing)
Height of laminar jet flame

Mixing layer analysis

\[ \frac{\partial \rho Z}{\partial t} + \nabla \cdot (\rho v Z) = \nabla \cdot (\rho D \nabla Z) \]

\[ L_f \approx \frac{v_f R^2}{D} \]
Height of laminar jet flame

Mixing layer analysis

\[ X = v_F t, \]
\[ Y = R - V_{O2} t = R - \frac{1}{\ell} D_{O2} t, \]
\[ X = L_f = v_F t_f, \]
\[ Y = 0 = R - \frac{1}{\ell} D_{O2} t_f, \]
\[ L_f \sim \frac{v_F R \ell}{D} \sim \frac{v_F R^2}{D}. \]

\[ Y_i V_i = D_i \nabla Y_i, \quad V_i = D_i / Y_i \nabla Y_i \propto D_i / \ell. \]
Jet diffusion flame: experimental observation

\[ L_f \sim \frac{v_F R^2}{D} = \left( \frac{v_F R}{D} \right) R = Pe \cdot R \]
Propagation of laminar diffusion flame

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \vec{v} = 0
\]

\[
\frac{\partial \rho Y_i}{\partial t} + \nabla \cdot \rho Y_i \vec{v} = \nabla \cdot \rho D_i \nabla Y_i + \omega_i
\]

\[
\frac{\partial \rho \vec{v}}{\partial t} + \nabla \rho \vec{v} = \nabla \cdot (p \mathbf{I} + \tau)
\]

\[
\rho \frac{Dh}{Dt} - \frac{Dp}{Dt} = \nabla \left( \rho \alpha \nabla h - \rho \alpha \sum_{i=1}^{N} \left( \frac{1}{1 - \frac{1}{L_{ei}}} \right) h_i \nabla Y_i \right) + \dot{Q} + \tau \cdot \nabla \vec{v}
\]
Burke-Schumann flame-sheet model

\[
\begin{align*}
\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \vec{v} &= 0 \\
\frac{\partial \rho Z}{\partial t} + \nabla \cdot (\rho \vec{v} Z) &= \nabla \cdot (\rho \alpha \nabla Z) \\
\frac{\partial \rho \vec{v}}{\partial t} + \nabla \rho \vec{v} &= \nabla \cdot (pI + \tau) \\
\rho \frac{Dh}{Dt} - \frac{Dp}{Dt} &= \nabla \cdot \left( \rho \alpha \nabla h - \rho \alpha \sum_{i=1}^{N} \left( 1 - \frac{1}{Le_i} \right) h_i \nabla Y_i \right) + \dot{Q} + \tau : \nabla \vec{v}
\end{align*}
\]

\[Y_i = a + bZ\]

\[T = c + dZ\]
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• Burke-Schumann flame-sheet model
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Finite-rate chemistry effect

- Burke-Schumann flame-sheet model is not always true; it is true only when the chemical reactions are much faster than the mixing of fuel and the oxidizer.

- If the mixing is very fast, what is going to happen?
Flamelet equation

Flamelet assumption:

\[ Y_i(x,y,z,t) = Y_i(Z(x,y,z,t)), \]
\[ \rho(x,y,z,t) = \rho(Z(x,y,z,t)), \]
\[ T(x,y,z,t) = T(Z(x,y,z,t)) \]

\[ \frac{\partial \rho Y_i}{\partial t} + \nabla \cdot (\rho Y_i \bar{v}) = \nabla \cdot (\rho D_i \nabla Y_i) + \omega_i \]

Species transport

Mixture fraction

\[ \frac{\partial \rho Z}{\partial t} + \nabla \cdot (\rho \bar{v}Z) = \nabla \cdot (\rho D \nabla Z) \]
Flamelet equation

\[-\frac{1}{2} \rho \chi \frac{d^2 Y_i}{dZ^2} = \omega_i\]

\[\frac{d^2 Y_i}{dZ^2} = -\frac{2\omega_i}{\rho \chi}\]

\[\chi = 2D(\nabla Z \cdot \nabla Z)\quad \text{Scalar dissipation rate: mixing rate}\]

Damköhler number \(Da = \frac{\text{mixing time}}{\text{chemical reaction time}}\)

\[Da \sim \frac{2\omega_i}{\rho \chi}\]
Finite rate chemistry effect

\[ \frac{d^2Y_i}{dZ^2} \rightarrow \infty \]

\[ \frac{d^2Y_i}{dZ^2} = \text{finite} \]

Damkohler number infinity

Damkohler not very high
Increase of scalar dissipation rate leads to

- the leakage of fuel to the oxygen rich side increases
- the leakage of oxygen to the fuel rich side increases
- the flame temperature decreases, as a result of insufficient oxidation of the fuel
Flame quenching at high scalar dissipation rate

\[ \chi_P^{-1} \left[ S \right] \]

- Quenched
- Flamelet
- Flame sheet

With preheating

Adiabatic flame

With radiation heat loss

\[ T_P \]

\[ T_q \]

\[ \chi_q^{-1} \]
Laminar jet diffusion flame

- Rich premixed flame
- Lean premixed flame
- Lift-off height
- Diffusion flame
- Burnable mixture

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Effect of mixing rate

- For very small scalar dissipation rate the flame temperature is high; changing the scalar dissipation a little would not affect the flame temperature. This regime can be described by the simple Burke-Schumann flame sheet model.

- For intermediate scalar dissipation rate, the flame temperature is lower than the Burke-Schumann flame temperature. Increasing the scalar dissipation rate leads to a decrease of the flame temperature until to a critical condition, at which the flame temperature changes rapidly. This regime may be called flamelet.

- For scalar dissipation rate higher than the quenching scalar dissipation rate, no flame exists. The flame is quenched.