Tissue Optical Properties

Indirect Measurements in Bulk Tissue

Medical Optics

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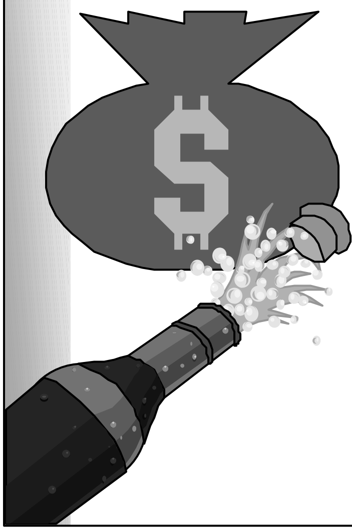
Evaluation with diffusion theory

Indirect measurements means that one measure some parameters and extract the properties sought by applying some type of theoretical model.

Ex: R_{tot}, R(r) **--->** μ_a, μ_s'

Inverse problem



The inverse problem is when you know the answer but not how to reach it!

Example in tissue optics:

You can measure diffuse reflectance, but what path did the photons take?

Methods Available

- Steady state methods
  - Added-absorber
  - Fluence rate vs. distance from source
  - Steady-state diffuse reflectance
- Time-resolved methods
  - Time-resolved diffuse reflectance
  - Frequency-domain diffuse reflectance

Steady State Diffusion

The steady state diffusion for a point source is

$$-\nabla^2 f(\mathbf{r}) + m_{eff}{}^2 f(\mathbf{r}) = S(\mathbf{r})$$

This yields a solution for a infinite homogenous medium as:

$$f(\mathbf{r}) = f(\mathbf{r}=\mathbf{0})\frac{1}{|\mathbf{r}|}\exp(-m_{eff}\cdot|\mathbf{r}|) = \boxed{\frac{Pm_{eff}{}^2}{4pm_a}\frac{1}{|\mathbf{r}|}\exp(-m_{eff}\cdot|\mathbf{r}|)}$$

Solution to steady state diffusion

$$f(\mathbf{r}) = \frac{Pm_{eff}{}^2}{4pm_a}\frac{1}{|\mathbf{r}|}\exp(-m_{eff}\cdot|\mathbf{r}|)$$

where

$$\begin{cases} m_{eff} = \sqrt{3m_a(m_a + m_s(1-g))} \\ m_s' = m_s(1-g) \\ D = \dfrac{1}{3(m_a + m_s(1-g))} = \dfrac{m_a}{m_{eff}{}^2} \end{cases}$$

Steady-state methods

- This means that it is primarily μ_{eff} that can be extracted from steady state diffusion from bulk tissue
- However, this is often what is required for light wavelengths obeying the diffusion approximation
- To be able to extract μ_a and μ_s' separately one need to measure more than the dependence of fluence rate vs. distance
- The g-value can never be evaluated with diffusion theory.

Direct Fluence Rate Measurements

The most used technique to evaluate μ_{eff} from steady state measurements is to directly measure the fluence rate.

To do that one need to be inside the tissue ==> invasive measurements.

Need to measure light coming from all directions ==> diffusing fibre tips.

$$f(\mathbf{r},t) = \int_{4p} L(\mathbf{r},\mathbf{s},t)\,dw$$

Steady State Diffuse Reflectance

This method is an alternative, non-invasive way to measure the r-dependence of the fluence rate.

By measuring at a boundary, one need to take the boundary conditions into account.

Calculate the steady state diffuse reflectance !

As an exercise, I would like that you derive an expression for the diffuse reflectance R(ρ) of a semi-infinite highly scattering medium.

The fluence rate inside the medium can be written as:

$$\varphi(r) = \frac{P\mu_{eff}^{\;2}}{4\pi\mu_a}\left\{\frac{\exp(-\mu_{eff}\cdot r_1)}{r_1} - \frac{\exp(-\mu_{eff}\cdot r_2)}{r_2}\right\}$$

where $\begin{cases} r_1 = \sqrt{(z - z_0)^2 + \rho^2} \\ r_2 = \sqrt{(z + z_0 + 2z_b)^2 + \rho^2} \end{cases}$

Ex: Guidelines (I)

The diffuse reflectance is derived as the photon density current at z = 0 perpendicular to the surface:

$$R(r) = hn\mathbf{J}(r,0)\cdot\mathbf{n} = -D\frac{\P}{\P z}f(r,z)\Big|_{z=0}$$

Ex: Guidelines (II)

$$f = f_1 + f_2$$

$$\frac{\P f}{\P z} = \sum_i \frac{\P f_i}{\P r_i}\frac{\P r_i}{\P z}$$

$$\frac{\P r_1}{\P z}\Big|_{z=0} = \frac{1}{2}\frac{1}{r_1}2(z - z_0) = -\frac{z_0}{r_1}$$

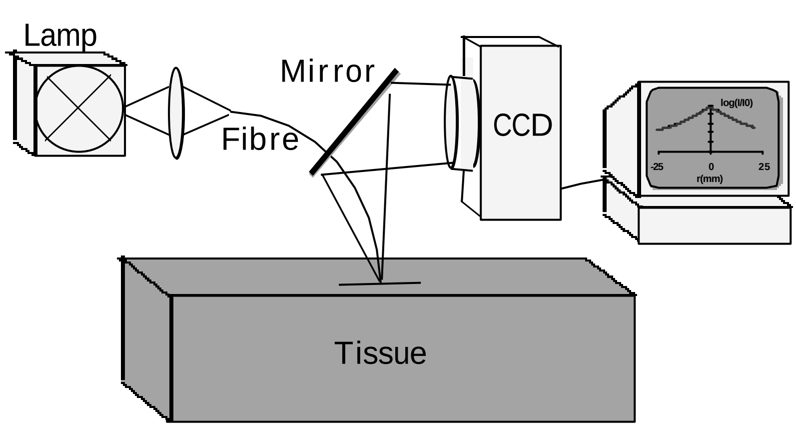
$$\frac{\P f_i}{\P r_i} = \left(-m_{eff} - \frac{1}{r_i}\right)f_i$$

Diffuse reflectance equation

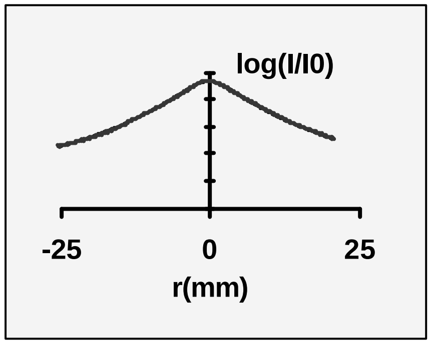
$$R(r) = \frac{1}{4p}\left[z_0 \left(m_{eff} + \frac{1}{r_1} \right) \frac{e^{-m_{eff} r_1}}{r_1^{\,2}} + (z_0 + 2z_b)\left(m_{eff} + \frac{1}{r_2} \right) \frac{e^{-m_{eff} r_2}}{r_2^{\,2}} \right]$$

- where $\mathbf{r_1}$ is the distances from the real source,
- $\mathbf{r_2}$ is the distance from the virtual source,
- $\mathbf{z_0}$ is the distances between the real source and the real boundary,
- $\mathbf{z_b}$ is the distance between the real boundary and the extrapolated boundary,
- $\mathbf{2z_b + z_0}$ is the distance between the real boundary and the virtual source.

Set-up for spatially resolved diffuse reflectance



Diffuse Reflectance from Skin at 630 nm



Diffuse reflectance of normal skin irradiated with a pencil beam of light at 630 nm, measured as line across the irradiated spot.

Added Absorber Method

Further another method that can be used to evaluate both μ_a and μ_s' is the so called added absorber method.

This method is destructive and can thus not be used *in vivo.* However, it is a very useful method for liquid tissue phantoms and for homogenised tissue.

The principle is to measure $\phi(r)$ vs. r, add some absorber and redo the measurements etc.

Added Absorber Theory

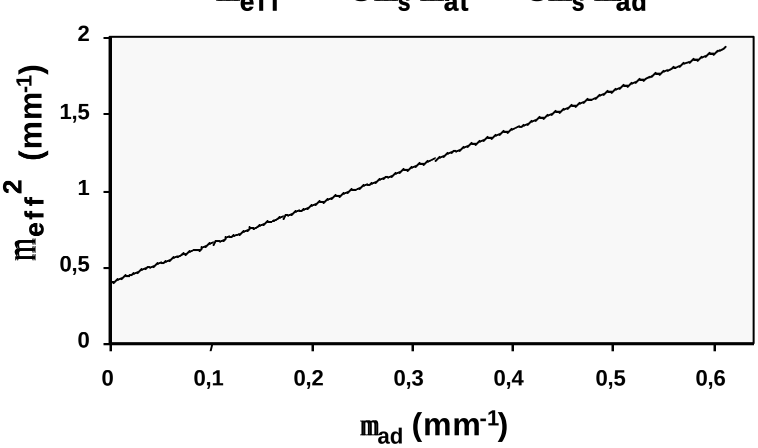
The aim is to measure μ_{eff} for a number of added absorber concentrations.

$$\mu_{eff}^{\;2} = 3\mu_a(\mu_a + \mu_s')$$

with some added absorption $\mu_a = \mu_{at} + \mu_{ad}$ for the tissue and added dye, respectively. If $\mu_{at} + \mu_{ad} \langle\langle \mu_s'$ then $\mu_{eff}^{\;2} \approx 3(\mu_{at} + \mu_{ad})\mu_s'$

Added Absorber Plot

$$\mu_{eff}^{\;2} = 3\mu_s'\mu_{at} + 3\mu_s'\mu_{ad}$$



Time-resolved diffuse reflectance

The diffuse fluence rate in an infinite medium using a point source δ(r=0,t=0) can be expressed as:

$$\phi(\mathbf{r},t) = c(4\pi Dct)^{-3/2} \exp(-\mu_a ct) \exp(-\frac{r^2}{4Dct})$$

Transform to frequency domain

The Fourier transform of this expression can be used to derive the solution for a modulated point source:

The fluence rate can be shown to be

$$S(\mathbf{r},t) = \delta(r=0)\left\{1 + M_s e^{-i\omega t}\right\}$$

$$\phi(\mathbf{r},t) = \frac{1}{4\pi Dr}\left[e^{-r\sqrt{\frac{\mu_a}{D}}} + M_s e^{\left[\frac{\mu_a}{D} + i\frac{\omega}{Dc}\right]^{1/2}} e^{-i\omega t} \right];\ with\quad \mu_{eff} = \sqrt{\frac{\mu_a}{D}}$$

This means both a phase shift and a demodulation

Diffusion equation in discrete form

The time resolved diffusion equation is

$$\frac{1}{c}\frac{\partial}{\partial t} f(\mathbf{r},t) - D\nabla^2 f(\mathbf{r},t) + m_a f(\mathbf{r},t) = S(\mathbf{r},t)$$

This can be rewritten in discrete form as

$$\frac{f_x^{t+1} - f_x^{t}}{\Delta t} = c'\frac{D}{2}\left[\frac{(f_{x+1}^{t+1} - 2f_x^{t+1} + f_{x-1}^{t+1}) + (f_{x+1}^{t} - 2f_x^{t} + f_{x-1}^{t})}{(\Delta x)^2}\right]$$

$$-c'\frac{m_a}{2}(f_x^t + f_x^{t+1})$$