Bed Load and Suspended Load. Sediment Transport Formulas

Environmental Hydraulics

Sediment Transport Modes

- bed load
  - along the bottom; particles in contact; bottom shear stress important
- suspended load
  - in the water column; particles sustained by turbulence; concentration profiles develop

Increasing Shields number
**Suspended Load**

Settling velocity less than upward turbulent component of velocity (for grains to remain in suspension).

Important parameter: \( \frac{w_s}{u^*} \).

\[
q_{ss} = \int_{z_a}^{h} c(z) u(z) dz
\]

**Sediment Concentration Profile**

Balance between sediment settling and upward sediment diffusion from turbulence:

\[
w_s C = -K_s \frac{dC}{dz}
\]

\[
C(z) = C_a \exp \left( -\int_{z_a}^{z} \frac{w_s}{K_s} dz \right)
\]
Sediment Diffusivity

Different expression for the diffusivity:

\[ K_s = K_o \quad \text{Constant} \]

\[ K_s = \kappa u_\varepsilon z \quad \text{Linear} \]

\[ K_s = \kappa u_\varepsilon z \left(1 - \frac{z}{h}\right) \quad \text{Parabolic} \]

Suspended Sediment Concentration Profiles

Exponential (constant diffusivity):

\[ C(z) = C_R \exp \left(-\frac{w_s}{K_o} z\right) \]

if \( w_s/K_o > 4 \): weak suspension

if \( w_s/K_o < 0.5 \): strong suspension
Power-law (linear diffusivity):

\[ C(z) = C_a \left( \frac{z}{z_a} \right)^{-w_s/\kappa u} \]

Rouse number (suspension parameter):

\[ b = \frac{w_s}{\kappa u} \]

- \( b > 5 \): near bed suspension (h/10)
- \( 5 > b > 2 \): suspension through bottom half of boundary layer
- \( 2 > b > 1 \): suspension throughout boundary layer
- \( 1 > b \): uniform suspension throughout boundary layer

Power-law (parabolic diffusivity):

\[ C(z) = C_a \left( \frac{z}{h-z} \frac{h-z_a}{z_a} \right)^{-w_s/\kappa u} \]  

(Rouse profile)

For power-law profiles \( z_a \) is an additional parameter to estimate besides \( C_a \).

More complicated diffusivity relationships exist (e.g., Van Rijn).

\[ => \text{More complicated concentration profiles.} \]
Comparison between concentration profiles

Different profiles

Rouse profile

Comparison with Data
(Camenen and Larson 2007)
Comparison with Data

Exponential Power-law (linear)

Similar fit for all concentration profiles (Camenen and Larson 2007)

Rouse profile

Settling Velocity

Depends on:
- particle diameter
- particle density
- particle concentration
- particle shape
- viscosity of water (temperature)
- turbulence

Dimensionless grain size for characterization of settling velocity:

\[ D_* = \left( \frac{g(s-1)}{\nu^2} \right)^{1/3} \ d_{50} \]
Settling Velocity

Soulsby (1997):

\[ w_s = \frac{v}{d} \left( \sqrt{10.36^2 + 1.049D^3} - 10.36 \right) \]

Reference Concentration and Height

Smith and McLean (1977) (power-law/linear):

\[
\begin{align*}
C_o &= \frac{0.0156T_s}{1 + 0.0024T_s} \\
T_s &= \frac{\tau_{os} - \tau_{cr}}{\tau_{cr}} \\
z_o &= \frac{26.3\tau_{cr}T_s}{\rho g(s - 1)} + \frac{d_{so}}{12}
\end{align*}
\]
**Suspended Load Transport**

Integrate product between concentration and velocity over the vertical.

For the exponential concentration profile and constant velocity:

$$q_{ss} = U_c c_R K_c \left[1 - \exp \left(-\frac{w_s h}{K_c}\right)\right]$$

Reference concentration (Camenen and Larson 2007):

$$c_R = A_{cR} \theta \exp \left(-4.5 \frac{\theta}{\theta_0}\right)$$

$$A_{cR} = 3.5 \cdot 10^{-3} \exp(-0.3D_r)$$

**Bed Load**

Threshold of motion exceeded ($\tau_o - \tau_{cr} > 0$) => sediment movement along bottom as bed load.

Rolling, sliding, and hopping (saltation) of grains along the bed.

Weight of the grains is borne by contact with other grains.

Bed load occurs:

- over flat beds at low flows
- in conjunction with ripples for stronger flows
- over a flat bed for very strong flows (sheet flow)
Bed load dominates for low flows and/or large grains.

Parameters to characterize bed load:

\[ \Phi = \frac{q_{sb}}{\sqrt{(s-1) g d_{50}^3}} \]

Dimensionless transport number

\[ \theta = \frac{\tau_o}{(\rho_s - \rho) g d_{50}} \]

Shields number

Bed Load Transport Formulas

Meyer-Peter and Müller (1948):

\[ \Phi = 8(\theta - \theta_{cr})^{3/2} \]

Nielsen (1992):

\[ \Phi = 12 \theta^{1/2} (\theta - \theta_{cr}) \]

Camenen and Larson (2006):

\[ \Phi = 12 \theta^{3/2} \exp\left(-4.5 \frac{\theta_{cr}}{\theta}\right) \]
Total Load Transport

Add bed load and suspended load => total load

Or: Predict bed load and suspended load at the same time (one formula for both transport modes).

Resolves the physics to a lesser degree, but practical.

Distinction between bed load and suspended load often hard to make.

Example of such total load formulas:

• Engelund-Hansen (1972)
• Ackers-White (1973)

(based on flow velocity)
Example: Engelund-Hansen total load formula

\[ q_t = \frac{0.05C_h^{3/2}U^5}{(g(s-1))^2 d_{50}} \]

Comparison between EH, VR, and AW formulas