Basic Sediment Transport and Boundary Layer Theory

Environmental Hydraulics

Sediment Transport

Interaction between fluid flow (water or air) and its loose boundaries.

Strong interaction between the flow and its boundaries.

Professor H.A. Einstein: "my father had an early interest in sediment transport and river mechanics, but after careful thought opted for the simpler aspects of physics"
Overview

- introduction
- properties of sediment
- current boundary layers
- threshold of motion
- bed features
- suspended load transport
- bed load transport
- total load transport

Importance of Sediment Transport

- erosion (land, rivers, coast)
  Examples: soil erosion, local scour around structures, beach erosion

- accumulation (rivers, lakes, reservoirs coast)
  Examples: reservoir sedimentation, infilling of harbors, navigation channels, and water intakes
Classification of Sediment

• cohesive (clay)
  electro-chemical forces; form a coherent mass
• non-cohesive (sand)
  frictional forces; collection of individual particles
Agents for Sediment Transport

- wind
- water
  - uni-directional flow (rivers)
  - oscillatory flow (waves)
  - combined flow (waves + current)

Properties of Sediment

- grain size (diameter)
- density
- porosity
- concentration (by volume or mass)
- angle of repose
- permeability (fluidization)
Grain-Size Classification

**Phi scale:**

\[ \varphi = -\log_2 d \]

Grain-Size Fractions

\[ d_n = \text{the grain diameter for which} \ n\% \text{ of} \ \text{the grains by mass is finer} \]

Geometric standard deviation:

\[ \sigma_g = \sqrt{d_{84}/d_{16}} \]
Sediment Sorting

\[ d_{84} / d_{16} < 2 \quad \text{well sorted} \]
\[ d_{84} / d_{16} > 16 \quad \text{well mixed} \]

Grain-size distribution often follows a log-normal distribution

Density, Concentration and Porosity

Table 2. Measures for sand-water mixtures

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Usage</th>
<th>Definition</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volume concentration</td>
<td>Suspension (theory)</td>
<td>Volume of mixture</td>
<td>( C )</td>
</tr>
<tr>
<td>Packing density</td>
<td>Bed (well sorted)</td>
<td>Volume of mixture</td>
<td>( e )</td>
</tr>
<tr>
<td>Dry density</td>
<td>Bed (well sorted)</td>
<td>Volume of mixture</td>
<td>( e )</td>
</tr>
<tr>
<td>Moisture density</td>
<td>Bed (well sorted)</td>
<td>Volume of mixture</td>
<td>( e )</td>
</tr>
<tr>
<td>Voids ratio</td>
<td>Bed (well sorted)</td>
<td>Volume of mixture</td>
<td>( e )</td>
</tr>
<tr>
<td>Suspension density</td>
<td>Bed (well sorted)</td>
<td>Volume of mixture</td>
<td>( e )</td>
</tr>
<tr>
<td>Salt density</td>
<td>Bed (well sorted)</td>
<td>Volume of mixture</td>
<td>( e )</td>
</tr>
</tbody>
</table>

Table 3. Porosity of natural sand hills

<table>
<thead>
<tr>
<th>Porosity</th>
<th>Well sorted</th>
<th>Average</th>
<th>Well mixed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loosest packed</td>
<td>0.46</td>
<td>0.42</td>
<td>0.70</td>
</tr>
<tr>
<td>Densest packed</td>
<td>0.40</td>
<td>0.37</td>
<td>0.50</td>
</tr>
</tbody>
</table>

Density of sand grains (quartz): 2650 kg/m³
**Bulk Density**

\[ \rho_B = \frac{\text{mass of mixture}}{\text{volume of mixture}} \]

**Angle of Respose**

The angle to the horizontal at which grains start to roll on a flat bed of sediment that is gradually tilted from the horizontal.

A representative value on the angle of repose is 32 deg.
Permeability and Fluidization

Bulk velocity:

\[ V_B = \frac{K_p}{\rho v} \frac{dp}{dz} \]

Current Boundary Layers

Velocity profile over a loose boundary (bed).
Vertical gradients are much larger than horizontal ones.

Logarithmic velocity profiles:

\[ U(z) = \frac{u_*}{\kappa} \ln \frac{z}{z_o} \]

\[ u_* = \frac{\tau_o}{\sqrt{\rho}} \]

Von Karman's constant
\[ \kappa = 0.4 \]

Mean velocity:
\[ \bar{U} = \frac{1}{h} \int_0^h U(z) dz \]
**Bed Roughness Length**

\[ z_o = \frac{v}{9u_s} \quad \text{Smooth flow} \]

\[ z_o = \frac{k_s}{30} + \frac{v}{9u_s} \quad \text{Transitional flow} \]

\[ z_o = \frac{k_s}{30} \quad \frac{u.k_s}{v} > 70 \quad \text{Rough flow} \]

Nikuradse roughness:

\[ k_s = 2.5d_{50} \]

**Current Skin Friction Shear Stress (Flat Bed)**

Shear stress:

\[ \tau_o = \rho C_D U^2 \]

Drag coefficient:

\[ C_D = \left( \frac{\kappa}{1 + \ln(z_o/h)} \right)^2 \]

Relationship with other friction coefficients:

\[ C_D = \frac{f}{8} = \frac{g}{C^2} = \frac{gn^2}{h^{1/3}} \]

( Darcy-Weisbach / Chezy / Manning )
Current Total Shear Stress (Non-Flat Bed)

Shear stress from skin friction and form drag:

\[ \tau_o = \tau_{os} + \tau_{of} \]

High flow speed \(\Rightarrow\) sheet flow \(\Rightarrow\) additional roughness

\[ z_o = z_{os} + z_{of} + z_{ot} \]

Wilson (1989):

\[ z_{ot} = \frac{5}{30} \frac{\tau_{os}}{g(\rho_s - \rho)} \]

Threshold of Motion

Conditions for initiation of motion.

\[ \frac{\tau_{cr}}{\rho g(s-1)d_{50}} = f\left(\frac{u.d_{50}}{v}\right) \]

(Shields 1936)

Shields diagram
Threshold current speed (Soulsby 1997):

\[ \mathcal{U}_{cr} = 7 \left( \frac{h}{d_{50}} \right)^{1/7} (g(s - 1)d_{50} f(D_c))^{1/2} \]

\[ f(D_c) = \frac{0.30}{1 + 1.2D_c} + 0.055(1 - \exp(-0.020D_c)) \]

\[ D_c = \left( \frac{g(s - 1)}{\nu^2} \right)^{1/3} d_{50} \]

---

Threshold bed shear stress (Soulsby and Whitehouse 1997):

\[ \theta_{cr} = \frac{0.30}{1 + 1.2D_c} + 0.055(1 - \exp(-0.020D_c)) \]

---

Threshold Shields parameter:

\[ \theta_{cr} = \frac{\tau_{cr}}{g(\rho_s - \rho)d_{50}} \]
Bed Features

A variety of features appears on a loose bed exposed to flowing water:

- ripples
- dunes
- antidunes

Example of Bed Features

(a) Current

(b) Current

(C) Current

(D)
Characterization of Bed Forms

Current Ripples

Geometric properties (Soulsby 1997):

\[ \lambda_r = 1000d_{50} \]  
Ripple wavelength

\[ \Delta_r = \frac{\lambda_r}{7} \]  
Ripple height
**Dunes**

Geometric properties (Soulsby 1997):

\[ \lambda_d = 7h \]

Dune wavelength

\[ \Delta_d = 0.07 \lambda_d \]

Dune height

(more complex formulas exist)

**Friction due to Bed Forms**

Skin friction and form drag contributes.

Form drag given by (Soulsby 1997):

\[ z_{w_f} = a_r \frac{\Delta^2}{\lambda_r} \]

\( a_r \), typically about 1.0