Motivation

Often difficult to solve fluid flow problems by analytical or numerical methods. Also, data are required for validation.

The need for experiments

Difficult to do experiment at the true size (prototype), so they are typically carried out at another scale (model).

Develop rules for design of experiments and interpretation of measurement results.
Fields of Application

• aerodynamics
• naval architecture
• flow machinery (pumps, turbines)
• hydraulic structures
• rivers, estuaries
• sediment transport

To solve practical problems, derive general relationships, obtain data for comparison with mathematical models.

Example of Model Experiments

Wind-tunnel
Towing tank
Spillway design
Sediment transport facility
Model Experiments in Hydraulics I

Construction of model

Model Experiments in Hydraulics II

Model of dam with spillways

Discharge at model spillway
Model Experiments in Hydraulics III

Traryd hydropower station

Model gates

Model Experiments in Hydraulics IV

pump intake

surge chamber

Hydraulic arrangements
Model Experiments in Hydraulics V

Lilla Edet

Water power plants

Lule Ålv

Terminology

**Similitude**: how to carry out model tests and how to transfer model results to prototype (*laws of similarity*)

**Dimensional analysis**: how to describe physical relationships in an efficient, general way so that the extent of necessary experiments is minimized (*Buckingham’s II-theorem*)
**Example: Drag Force on an Submerged Body**

Drag force ($D$) depends on:
- diameter ($d$)
- velocity ($V$)
- viscosity ($\mu$)
- density ($\rho$)

$$D = f(d, V, \mu, \rho)$$

Dimensional analysis:
$$\frac{D}{\rho V^2 d^2} = f\left(\frac{V d \rho}{\mu}\right) = f(Re)$$

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**Basic Types of Similitude**

- geometric
- kinematic
- dynamic

All of these must be obtained for complete similarity between model and prototype.
Geometric Similarity

Ratios between corresponding lengths in model and prototype should be the same.

\[ \frac{d_p}{d_m} = \frac{l_p}{l_m} = \lambda \]

\[ \frac{A_p}{A_m} = \left( \frac{d_p}{d_m} \right)^2 = \left( \frac{l_p}{l_m} \right)^2 = \lambda^2 \]

Kinematic Similarity

Flow field in prototype and model have the same shape and the ratios of corresponding velocities and accelerations are the same.

\[ \frac{V_{1p}}{V_{1m}} = \frac{V_{2p}}{V_{2m}}, \quad \frac{a_{1p}}{a_{1m}} = \frac{a_{2p}}{a_{2m}} \]

Geometrically similar streamlines are kinematically similar.
Dynamic Similarity

To ensure geometric and kinematic similarity, dynamic similarity must also be fulfilled.

Ratio between forces in prototype and model must be constant:

\[
\frac{F_{1p}}{F_{1m}} = \frac{F_{2p}}{F_{2m}} = \frac{F_{3p}}{F_{3m}} = \frac{M_p a_p}{M_m a_m}
\]  
(vector relationships)

Also, Newton’s second law:

\[
F_{1p} + F_{2p} + F_{3p} = M_p a_p
\]
\[
F_{1m} + F_{2m} + F_{3m} = M_m a_m
\]

Important Forces for the Flow Field

- pressure \((F_p)\)
- inertia \((F_i)\)
- gravity \((F_g)\)
- viscosity \((F_v)\)
- elasticity \((F_e)\)
- surface tension \((F_t)\)
Parameterization of Forces

\[ F_p = \Delta p A = \Delta p l^2 \]
\[ F_l = M a = \rho l^3 \frac{V^2}{l} = \rho V^2 l^2 \]
\[ F_G = M g = \rho l^3 g \]
\[ F_r = \mu \frac{dV}{dy} A = \mu \frac{V}{l} l^2 = \mu V l \]
\[ F_T = \sigma l \]
\[ F_E = EA = El^2 \]

(convective acceleration: \( V \frac{dV}{dx} \))

Dynamic similarity: corresponding force ratios the same in prototype and model

\[
\begin{align*}
\left( \frac{F_l}{F_p} \right)_{p} &= \left( \frac{F_l}{F_p} \right)_{m} = \left( \rho \frac{V^2}{\Delta p} \right)_{p} = \left( \rho \frac{V^2}{\Delta p} \right)_{m} \\
\left( \frac{F_l}{F_V} \right)_{p} &= \left( \frac{F_l}{F_V} \right)_{m} = \left( \frac{V l p}{\mu} \right)_{p} = \left( \frac{V l p}{\mu} \right)_{m} \\
\left( \frac{F_l}{F_G} \right)_{p} &= \left( \frac{F_l}{F_G} \right)_{m} = \left( \frac{V^2}{g l} \right)_{p} = \left( \frac{V l p}{\mu} \right)_{m} \\
\left( \frac{F_l}{F_E} \right)_{p} &= \left( \frac{F_l}{F_E} \right)_{m} = \left( \frac{\rho V^2}{E} \right)_{p} = \left( \frac{\rho V^2}{E} \right)_{m} \\
\left( \frac{F_l}{F_T} \right)_{p} &= \left( \frac{F_l}{F_T} \right)_{m} = \left( \frac{\rho l V^2}{\sigma} \right)_{p} = \left( \frac{\rho l V^2}{\sigma} \right)_{m}
\end{align*}
\]
Dimensionless Numbers

- **Reynolds**
  \[
  \text{Re} = \frac{\frac{Vl}{\mu / \rho}}{v} = \frac{Vl}{\nu}
  \]

- **Froude**
  \[
  \text{Fr} = \frac{V}{\sqrt{gl}}
  \]

- **Cauchy (Mach)**
  \[
  C = \frac{\frac{V^2}{E / \rho}}{\frac{V^2}{c^2}} = M^2
  \]

- **Weber**
  \[
  W = \frac{\rho l V^2}{\sigma}
  \]

- **Euler**
  \[
  E = V \sqrt{\frac{\rho}{2 \Delta p}}
  \]

Dimensionless numbers same in prototype and model produces dynamic similarity.

Only four numbers are independent (fifth equation from Newton’s second law).

In most cases it is not necessary to ensure that four numbers are the same since:

- all forces do not act
- some forces are of negligible magnitude
- forces may counteract each other to reduce their effect
Reynolds Similarity I

Low-speed flow around air foil (incompressible flow)

\[ \left( \frac{Vl}{v} \right)_p = Re_p = Re_m = \left( \frac{Vl}{v} \right)_m \]

(also geometric similarity)

Same Re number yields same relative drag force:

\[ \left( \frac{D}{\rho V^2 I^2} \right)_p = \left( \frac{D}{\rho V^2 I^2} \right)_m \]

Reynolds Similarity II

Flow through a contraction (incompressible flow)

\[ \left( \frac{Vl}{v} \right)_p = Re_p = Re_m = \left( \frac{Vl}{v} \right)_m \]

(also geometric similarity)

Same Euler number yields same relative pressure drop:

\[ \left( \frac{\Delta p}{\rho V^2} \right)_p = \left( \frac{\Delta p}{\rho V^2} \right)_m \]
**Froude Similarity I**

Flow around a ship (free surface flow)

\[
\left( \frac{V}{\sqrt{gl}} \right)_p = Fr_p = Fr_m = \left( \frac{V}{\sqrt{gl}} \right)_m
\]

(frictional effects neglected)

Same Re number yields same relative drag force:

\[
\left( \frac{D}{\rho V^2 l^2} \right)_p = \left( \frac{D}{\rho V^2 l^2} \right)_m
\]

**Froude Similarity II**

Flow around a ship, including friction

\[
\left( \frac{V}{\sqrt{gl}} \right)_p = Fr_p = Fr_m = \left( \frac{V}{\sqrt{gl}} \right)_m
\]

\[
\left( \frac{Vl}{v} \right)_p = Re_p = Re_m = \left( \frac{Vl}{v} \right)_m
\]

Fulfill both: \( \frac{v_p}{v_m} = \left( \frac{l_p}{l_m} \right)^{3/2} \)

(typically not possible)
Reynolds Modeling Rules

Same viscosity in prototype and model

\[ \text{Re}_p = \text{Re}_m \rightarrow \frac{V_p}{V_m} = \frac{l_m}{l_p} = \lambda^{-1} \]

\[ \frac{V_p}{V_m} = \frac{l_p}{l_m} \rightarrow \frac{t_p}{t_m} = \lambda^2 \]

Similarly:

\[ \frac{Q_p}{Q_m} = \frac{I_p^3}{I_m^3} = \lambda^3 \frac{1}{\lambda^2} = \lambda \]

Froude Modeling Rules

Same acceleration due to gravity

\[ \text{Fr}_p = \text{Fr}_m \rightarrow \frac{V_p}{V_m} = \left( \frac{l_p}{l_m} \right)^{1/2} = \lambda^{1/2} \]

\[ \frac{V_p}{V_m} = \frac{l_p}{l_m} \rightarrow \frac{t_p}{t_m} = \lambda^{1/2} \]

Similarly:

\[ \frac{Q_p}{Q_m} = \frac{I_p^3}{I_m^3} = \lambda^3 \frac{1}{\sqrt{\lambda}} = \lambda^{5/2} \]
## Summary of Different Model Rules

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<th>Characteristic</th>
<th>Reservoir</th>
<th>Freestream</th>
<th>Hypodermic</th>
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### Kinematic properties

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### Dynamic properties

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### Distorted Scale

The vertical and horizontal scale is different (e.g., model of an estuary, bay, lagoon).

\[
\frac{l_p}{l_m} = \lambda_h, \quad \frac{y_p}{y_m} = \lambda_v,
\]

**Froude rule:**

\[
\frac{V_p}{V_m} = \left( \frac{y_p}{y_m} \right)^{1/2} = \lambda_v^{1/2}
\]

\[
\frac{V_p}{V_m} = \frac{l_p}{l_m} \quad \rightarrow \quad \frac{t_p}{t_m} = \frac{V_m l_p}{V_p l_m} = \frac{\lambda_h}{\sqrt{\lambda_v}}
\]
Movable-Bed Models

Model sediment transport and the effect on bed change:

• General river morphology
• River training
• Flood plain development
• Bridge pier location and design
• Local scour
• Pipeline crossings

Additional problem besides similarity for the flow motion is the similarity for the sediment transport and the interaction between flow and sediment.

Sediment Transport

Interaction between fluid flow (water or air) and its loose boundaries.

Strong interaction between the flow and its boundaries.

Professor H.A. Einstein: "my father had an early interest in sediment transport and river mechanics, but after careful thought opted for the simpler aspects of physics"
Movable-Bed Experiment (USDA)

Gravel

Cohesive material

Important Features of Sediment Transport

- initiation of motion
- bed features
- transport mode
- transport magnitude
- slope stability