Rapidly Varied Channel Flow

Example of such flows to be discussed:
• bridge pier contractions
• control of hydraulic jumps
• drop spillway structures
• channel transitions

Rapidly varied flows previously discussed
• broad- and sharp-crested weirs
• critical flow flumes

Discussion focuses on practical aspects rather than theoretical.
Bridge Piers

Bridge piers imply a constriction of the area. Backwater effects may occur => increased water level upstream with possible flooding.

Flow conditions at the pier:
• acceleration of the flow upstream
• continued acceleration and lowering of the water level at the constriction
• a vena contracta type section occurs in the constriction
• downstream the constriction uniform flow is re-established

Flow around the bridge pier might be analyzed using the momentum equation (takes into account the drag on the piers). However, an accurate solution typically requires experiments.

Formula developed by Yarnell (based on experiments):

\[
\frac{\Delta y}{y_3} = k Fr_3^2 \left( k + 5 Fr_3^2 - 0.6 \right) \left( \sigma + 15 \sigma^4 \right)
\]

\[
\sigma = 1 - \Gamma
\]

\[
\Gamma = \frac{b_2}{b_1}
\]

<table>
<thead>
<tr>
<th>Pier shape</th>
<th>k</th>
</tr>
</thead>
<tbody>
<tr>
<td>Semicircular nose and tail</td>
<td>0.9</td>
</tr>
<tr>
<td>Long-shaped nose and tail*</td>
<td>0.9</td>
</tr>
<tr>
<td>Twin-cylinder piers with connecting diaphragm</td>
<td>0.95</td>
</tr>
<tr>
<td>Twin-cylinder piers without diaphragm</td>
<td>1.05</td>
</tr>
<tr>
<td>50° triangular nose and tail</td>
<td>1.05</td>
</tr>
<tr>
<td>Square nose and tail</td>
<td>1.95</td>
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</tbody>
</table>

* A long-shaped nose or tail is formed from two circular curves each having a radius of twice the pier width and each tangential to a pier face.
Yarnell formula only valid as long as the flow is not choked (i.e., no backwater effects).

Limiting value on $\Gamma$ in order for choking not to take place may be calculated from ($E_1 = E_2$ assumed):

$$\Gamma_L = \left(\frac{27 Fr_i^2}{(2 + Fr_i^2)^3}\right)^{1/2}$$

If $\Gamma < \Gamma_L$ then choking occurs.

**Derivation of Choking Conditions**

The limit conditions corresponds to critical flow in the constriction and no backwater effects upstream.

Assume $E_1 = E_2$ and critical flow in section 2:

$$E_i = y_i + \frac{u_i^2}{2g} = \frac{3}{2} y_c = E_2$$

$$\frac{u_i^2}{2gy_i} + 1 = \frac{3 y_c}{2 y_i}$$

$$\frac{Fr_i^2}{2} + 1 = \frac{3 y_c}{2 y_i}$$

$$Fr_i^2 + 2 = \frac{3 y_c}{y_i}$$
Continuity equation:

\[ u_1 y_1 b_1 = u_c y_c b_2 \]
\[ u_1 y_1 b_1 = \sqrt{g y_c y_c} b_2 \]
\[ \frac{u_1}{\sqrt{g y_1}} b_1 = \sqrt{\frac{y_c y_c}{y_1 y_1}} b_2 \]
\[ Fr b_1 = \left( \frac{y_c}{y_1} \right)^{3/2} b_2 \quad \rightarrow \quad \frac{y_c}{y_1} = \frac{\left( \frac{Fr_1}{b_2 / b_1} \right)^{2/3}}{\Gamma} \]

Combine energy equation and continuity equation:

\[ Fr_1^2 + 2 = 3 \left( \frac{Fr_1}{\Gamma} \right)^{2/3} \]
\[ \left( \frac{Fr_1^2 + 2}{3} \right)^{3/2} = \frac{Fr_1}{\Gamma} \]
\[ \rightarrow \quad \Gamma = \left( \frac{27 Fr_1^2}{\left( Fr_1^2 + 2 \right)^3} \right)^{1/2} \]
Method by Matthai for Flow Estimation

Flow might be calculated from (compare previous discussion on flow measurements):

\[ Q = C_D A_2 \sqrt{2g \left( \Delta y + \alpha \frac{u_i^2}{2g} - h_j \right)} \]

\( C_D \) a function of many different variables.

Different Types of Bridge Openings I

Geometric properties of a bridge opening have a significant effect on the flow.

Type 1

Type 2
Different Types of Bridge Openings II

Control of Hydraulic Jump

If a hydraulic jump occurs on a smooth, horizontal surface there might be little variation in the upstream and downstream depths => The jump becomes fairly unstable (migrates easily).

However, there are different ways to fix the jump.
Example of structures used to fix a jump:
- sharp-crested weir
- broad-crested weir
- abrupt rise or drop
- stilling basin

Analytical approaches available, but normally have to resort to experiments.
**Dimensional Analysis of Control Structure**

The following relationship is obtained:

\[
\frac{\Delta z}{y_1} = \Theta \left( Fr_1, \frac{X}{y_2}, \frac{y_3}{y_1} \right)
\]

\(\Delta z\): height of the sill  
\(Fr_1\): Froude number in approaching flow  
\(y_1\): depth of approaching flow  
\(y_2\): depth of flow immediately upstream the sill  
\(y_3\): depth of flow downstream  
\(X\): distance from toe of jump to the sill

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**Sharp-Crested Weir**

Estimate the effect of the weir, if the upstream conditions are known (no effect from downstream depth).

Point above curves: jump moves upstream  
Point below curves: incomplete jump
**Broad-Crested Weir**

No tailwater effects if:

\[ y_3 < \frac{2y_2 + \Delta z}{3} \]

Analytic relationship between Fr, and \( \Delta z/y \), for jump control.

**Abrupt Rise**

Apply the momentum equation 1 → 2 and 2 → 3 + continuity equation
(results of such an analysis plotted below)
**Abrupt Drop**

If the tailwater depth is larger than the sequent depth a drop is needed to induce the jump.

Continuity equation and momentum equation used in the analysis.

Stable hydraulic jump might be achieved within certain parameter ranges.

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**Stilling Basin**

A short length of a paved channel placed at the end of a spillway (where supercritical flow occurs).

Construction details:

- **Chute block**: used to channelize and lift the flow (reduce the jump length)
- **Sill**: used to reduce the length of the jump and control scour
- **Baffle piers**: used to dissipate energy by impact
Spillway Design

Stilling Basin
Design Curves for USBR Basin II

FIGURE 9.32 Design curves and proportions of USBR basin II.
**Drop Spillway**

Drop spillways are used in small drainage structures to dissipate energy.

Aerated free-falling nappe followed by a jump.

**Example of a Drop Spillway**

Definition of primary variables.

Design guidelines available in French.
Transition Structures

Change in cross-sectional shape of the channel with the purpose of:

• minimize energy loss
• eliminate standing waves, turbulence etc
• provide safety