Theory of turbo machinery / Turbomaskinernas teori

Chapter 4
Axial-flow Turbines: 2-D theory

FIG. 4.1. Turbine stage velocity diagrams.

Note direction of $\alpha_2$
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Assumptions:

- Hub to tip ratio high (close to 1)
- Negligible radial velocities
- No changes in circumferential direction (wakes and nonuniform outlet velocity distribution neglected)
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Continuity equation for uniform steady flow:

$$\rho_1 A_1 c_{x1} = \rho_2 A_2 c_{x2} = \rho_3 A_3 c_{x3}$$

Assuming constant axial velocity

$$c_{x1} = c_{x2} = c_{x3} = c_x$$

$$\rho_1 A_1 = \rho_2 A_2 = \rho_3 A_3$$
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Work done on rotor by unit mass of fluid

$$\Delta W = \dot{W}/\dot{m} = h_{01} - h_{03} = U \left( c_{2y} + c_{3y} \right)$$

Please note: No work done in nozzle row:

$$h_{01} = h_{02}$$

With

$$h_0 = h + c^2/2 = \left( c_x^2 + c_y^2 \right)/2$$

And using above equations:

$$h_{02} - h_{03} = h_2 - h_3 + \left( c_x^2 + c_y^2 \right)/2 = U \left( c_{y2} + c_{y3} \right)$$
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Rewriting this in terms of relative velocity

\[ c_{y2} - U = w_{y2} \]
\[ c_{y3} + U = w_{y3} \]
\[ c_{y2} + c_{y3} = w_{y2} + w_{y3} \]

Combining above equations:

\[ h_2 - h_3 + \left( \frac{w_{y2}^2 + w_{y3}^2}{2} \right) = 0 \]

with \( w_{x2} = w_{x3} = c_x \) and \( w_x^2 + w_y^2 = w^2 \)

\[ h_2 - h_3 + \left( \frac{w_2^2 - w_3^2}{2} \right) = 0 \]

Relative stagnation enthalpy, \( h_{0,rel} \), does not change across rotor
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Nozzle row (1 to 2):

- Static pressure: \( p_1 \rightarrow p_2 \)
- Stagnation enthalpy: \( h_{01} = h_{02} \)
- Stagnation pressure:
  (isentropic: \( p_{01} = p_{02} \))

Subscript ‘s’ denotes isentropic change and ‘ss’ denotes both rows isentropic

FIG. 4.2. Mollier diagram for a turbine stage.
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Rotor row (2 to 3):
- Static pressure: \( h_{02} > h_{03} \)
- Stagnation enthalpy: \( p_{02} > p_{03} \)

However:
- Relative Stagnation enthalpy,
  \[ h_{02,\text{rel}} = h_{02} + \frac{w_2^2}{2} = h_{03,\text{rel}} \]

FIG. 4.2. Mollier diagram for a turbine stage.
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Turbine stage total to total efficiency:

\[ \eta_{tt} = \frac{\text{Actual work output}}{\text{Ideal work output when operating to same back pressure}} = \frac{h_{01} - h_{03}}{h_{01} - h_{03ss}} \]

For a normal stage, no changes in are made in velocities from inlet to outlet: \( c_1 = c_3 \) and \( \alpha_1 = \alpha_3 \). Further assuming \( c_{3ss} = c_3 \) the efficiency becomes:

\[ \eta_{tt} = \frac{h_{01} - h_{03}}{h_{01} - h_{03ss}} = \frac{h_1 - h_3}{h_1 - h_{3ss}} \]
### Axial-flow Turbines: 2-D theory

Defining enthalpy loss coefficients for the nozzle and rotor respectively:

\[\zeta_N = \frac{h_2 - h_{2s}}{c_2^2/2}\quad \text{and} \quad \zeta_R = \frac{h_3 - h_{3s}}{w_3^2/2}\]

Neglecting rotor temperature drop, the stage efficiencies may be expressed as:

\[\eta_u = \left[1 + \frac{\zeta_R w_3^2 + \zeta_N c_2^2}{2(h_1 - h_3)}\right]^{-1}\]

\[\eta_s = \left[1 + \frac{\zeta_R w_3^2 + \zeta_N c_2^2 + c_1^2}{2(h_1 - h_3)}\right]^{-1}\]
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Soderberg’s correlation:

- Large set of data compiled
- Design assuming Zweifel’s criteria for optimum space – axial chord ratio

\[ \Psi_r = \frac{Y}{Y_{id}} = 2 \left( \frac{s}{b} \right) \cos^2 \alpha_2 \left( \tan \alpha_1 + \tan \alpha_2 \right) \approx 0.8 \]

- **Result**: Turbine blade losses are a function of
  - Deflection \( \varepsilon \)
  - Blade aspect ratio \( H/b \)
  - Blade thickness-chord ratio \( t_{\text{max}}/l \)
  - Reynolds number
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- Deflection \( \varepsilon = \alpha_1 + \alpha_2 \)
- Blade aspect ratio: \( H/b = 3 \)
- Blade thickness-chord ratio \( t_{\text{max}}/l = 0.15 - 0.3 \)

\[
\text{Re} = \frac{\rho_2 c_2 D_h}{\mu} \quad D_h \text{ defined at exit throat} \quad D_h = \frac{2sH \cos \alpha_2}{(s \cos \alpha_2 + H)} = 10^5
\]

\( H \) is height of blade (radial direction)

\( l \)

\( b \)

\( c_t \)

\( \alpha_1 \)

\( t_{\text{max}} \)

Nozzle row
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For turbines:
- Deflection, $\varepsilon = \alpha_1 + \alpha_2$, is large, but $\alpha_2 \approx \alpha_2'$
- Deviation, $\delta = \alpha_2 - \alpha_2'$, is small $\varepsilon = \alpha_1' + \alpha_2'$
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FIG. 4.3. Soderberg’s correlation of turbine blade loss coefficient with fluid deflection (adapted from Horlock (1960).
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Corrections for

- Reynolds number  \( \text{Re} \neq 10^5 \)

\[
\zeta_{\text{cor}}^* = \left( \frac{10^5}{\text{Re}} \right)^{1/4} \zeta^*
\]

- Blade aspect ratio
  - Nozzles:  \( 1 + \zeta_{\text{cor}}^* = (1 + \zeta^*)(0.993 + 0.021 b/H) \)
  - Rotors:  \( 1 + \zeta_{\text{cor}}^* = (1 + \zeta^*)(0.975 + 0.075 b/H) \)

Tip clearance losses and disc friction not included
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Design considerations

• Rotor angular velocity (stresses, grid phasing)
• Weight (aircraft)
• Outside diameter (aircraft)
• Efficiency (almost always)
• ..........
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Consider a case with given

- Blade speed \( U \)
- Specific work \( \Delta W = U \left( c_{y2} + c_{y3} \right) \)
- Axial velocity \( c_x \)

The only remaining parameter to define is \( c_{y2} \) since

\[
c_{y3} = \frac{\Delta W}{U} - c_{y2}
\]

- Triangles may be constructed
- Loss coefficients determined from Soderberg
- Efficiencies computed from loss coefficients
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FIG. 4.4. Variation of efficiency with $c_{y2}/U$ for several values of stage loading factor $\Delta W/U^2$ (adapted from Shapiro et al. 1957).

Stage loading factor: $\frac{\Delta W}{U^2}$

flow coefficient: $\frac{c_x}{U}$

Aspect ratio: $\frac{H}{b}$
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Stage reaction, $R$

- Alternative description to $c_{y2}/U$
- Several definitions available
- Here: $R = (h_2 - h_3) / (h_1 - h_3)$

E.g: $R = 0.5$

$$0.5 = (h_2 - h_3) / (h_1 - h_3)$$

$h_2 - h_3 = h_1 - h_2$

\[ R = 0.5 \]
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For a normal stage, \( c_1 = c_3 \)

\[
R = \frac{h_2 - h_3}{(h_{01} - h_{03})}
\]

Using eq. 4.4: \( h_2 - h_3 + \left( \frac{w_2^2 - w_3^2}{2} \right) = 0 \) and Euler

\[
R = \frac{w_3^2 - w_2^2}{2U(c_{y2} + c_{y3})}
\]

\[
R = \frac{(w_3 - w_2)(w_3 + w_2)}{2U(c_{y2} + c_{y3})} = \frac{w_3 - w_2}{2U}
\]
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Relative tangential velocity

\[ w_y = c_x \tan \beta \]

\[ R = \frac{w_3 - w_2}{2U} = \frac{c_x}{2U} (\tan \beta_3 - \tan \beta_2) \]

Or using \[ c_{y2} = w_{y2} + U \]

\[ R = \frac{w_3 - w_2}{2U} = \frac{w_3 + U - w_{y2}}{2U} = \]

\[ = \frac{1}{2} + \frac{c_x}{2U} (\tan \beta_3 - \tan \alpha_2) \]
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Zero reaction stage

\[ R = \frac{c_s}{2U} (\tan \beta_3 - \tan \beta_2) = 0 \text{ if } \beta_3 = \beta_2 \]

FIG. 4.5. Velocity diagram and Mollier diagram for a zero reaction turbine stage.
50% reaction stage

\[ R = \frac{1}{2} + \frac{c_x}{2U} (\tan \beta_3 - \tan \alpha_2) = 0.5 \text{ if } \beta_3 = \alpha_2 \]

FIG. 4.7. Velocity diagram and Mollier diagram for a 50% reaction turbine stage.
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FIG. 4.8. Velocity diagram for 100% reaction turbine stage.
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**FIG. 4.4**

\[ R = 1 + \frac{\Delta W}{2U^2} - \frac{C_{y2}}{U} \]

**FIG. 4.9.** Influence of reaction on total-to-static efficiency with fixed values of stage loading factor.
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FIG. 4.6. Mollier diagram for an impulse turbine stage.
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Alternative representation for specified reaction:

\[ \eta = f(\Psi, \Phi) \]

where

\[ \Psi = \frac{\Delta W}{U^2} \]

is the stage loading and

\[ \Phi = \frac{c_x}{U} \]

is the flow coefficient.

FIG. 4.10. Design point total-to-total efficiency and deflection angle contours for a turbine stage of 50 percent reaction.
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FIG. 4.11. Design point total-to-total efficiency and rotor flow deflection angle for a zero reaction turbine stage.
Centrifugal stresses

\[ dF_c = -\Omega^2 r dm \]

\[ dm = \rho A dr \]

\[ \frac{d\sigma_c}{\rho} = \frac{dF_c}{\rho A} = -\Omega^2 r dr \]

With constant cross section this may be integrated

\[ \frac{\sigma_c}{\rho} = \Omega^2 \int_{r_t}^{r_h} r dr = \frac{U_{tip}^2}{2} \left[ 1 - \left( \frac{r_h}{r_t} \right) \right] \]

FIG. 4.15. Centrifugal forces acting on rotor blade element.
**Axial-flow Turbines: 2-D theory**

**Tapering:**
Reduction of cross sectional area in radial direction, in order to reduce stresses

Pure fluid dynamics would recommend the opposite

FIG. 4.16. Effect of tapering on centrifugal stress at blade root (adapted from Emmert 1950).
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FIG. 4.17. Maximum allowable stress for various alloys (1000 hr rupture life) (adapted from Freeman 1955).
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FIG. 4.18. Properties of Inconel 713 Cast (adapted from Balje 1981).
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Turbine blade cooling.

Why is the efficiency of the gas turbine comparable to that of a Rankine cycle?

(given that we do have to pay a considerable amount of energy to the compressor, whereas compression of water in the Rankine cycle is cheap)
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FIG. 4.20. Turbine thermal efficiency vs inlet gas temperature (adapted from le Grivès 1986).