Constraint Programming in Embedded System Design

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Outline

1. **Introduction to CP**
   - Sudoku Example
2. **Examples**
   - Geometrical Constraints
   - Scheduling
   - Set Constraints
3. **Graph Constraints in Emendded System Design**
   - Connected Componnets
   - (Sub-)graph Isomorphism

Programming with Constraints

Eugene C. Freuder
CONSTRAINTS, April 1997

"Constraint programming represents one of the closest approaches computer science has yet made to the Holy Grail of programming: the user states the problem, the computer solves it."

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Jean-François Puget, ILOG/IBM

Constraint Programming is Software Engineering applied to Operations Research

A simple definition

- Constraint programming is the study of computational systems based on constraints.
- The idea of constraint programming is to solve problems by exploring constraints which must be satisfied by the solution.

Example– SUDOKU

2	6		3					
5						7		
					1		4	
6			5			2		
		4			8			1
	5		9					
		7						3
					4		1	6

Example– SUDOKU

2	6		3					
5						7		
					1		4	
6			5			2		
		4			8			1
	5		9					
		7						3
					4		1	6

Helmut Simonis, IC-PARC, Imperial College London

Constraints programming has finally reached the masses, thousands of newspaper readers are solving their daily constraint problem.

Solution Method

2	6		3					
5						7		
					1		4	
6			5			2		
		4			8			1
	5		9					
		7						3
					4		1	6

Variables

```
v[i,j] :: {1..9}
```

Solution Method

2	6		3					
5						7		
					1		4	
6			5			2		
		4			8			1
	5		9					
		7						3
					4		1	6

Variables

```
v[i,j] :: {1..9}
```

Constraints

```
// Rows
v[1,1] ≠ v[1,2], ...
// Columns
v[1,1] ≠ v[2,1], ...
// Squares
v[1,1] ≠ v[2,2], ...
```

Solution Method

2	6		3					
5						7		
					1		4	
6			5			2		
		4			8			1
	5		9					
		7						3
					4		1	6

Values for first row after "simple" consistency

```
2 6 {1, 8..9}  3 {4..5, 7..9}  {5, 7, 9}
{1, 5, 8..9}  {5, 8..9}  {5, 8..9}
```

More advanced consistency

Values

```
2 6 {1, 8..9} 3 {4..5, 7..9}  {5, 7, 9}
{1, 5, 8..9} {5, 8..9} {5, 8..9}
```

- values 1, 5, 8 and 9 need to be assigned to variables v[1,3], v[1,7], v[1,8], and v[1,9],
- values 1, 5, 8 and 9 can be removed from other variables in this row.

More advanced consistency

Values

```
2 6 {1, 8..9} 3 {4..5, 7..9}  {5, 7, 9}
{1, 5, 8..9} {5, 8..9} {5, 8..9}
```

- values 1, 5, 8 and 9 need to be assigned to variables v[1,3], v[1,7], v[1,8], and v[1,9],
- values 1, 5, 8 and 9 can be removed from other variables in this row.

New values

```
2 6 {1, 8..9} 3 4 7
{1, 5, 8..9}  {5, 8..9}  {5, 8..9}
```

Solution

2	6	9	3	4	7	1	8	5
5	4	1	8	9	6	7	3	2
8	7	3	2	5	1	6	4	9
6	1	8	5	7	3	2	9	4
3	2	5	4	1	9	8	6	7
7	9	4	6	2	8	3	5	1
1	5	6	9	3	2	4	7	8
4	8	7	1	6	5	9	2	3
9	3	2	7	8	4	5	1	6

SUDOKU in MiniZinc

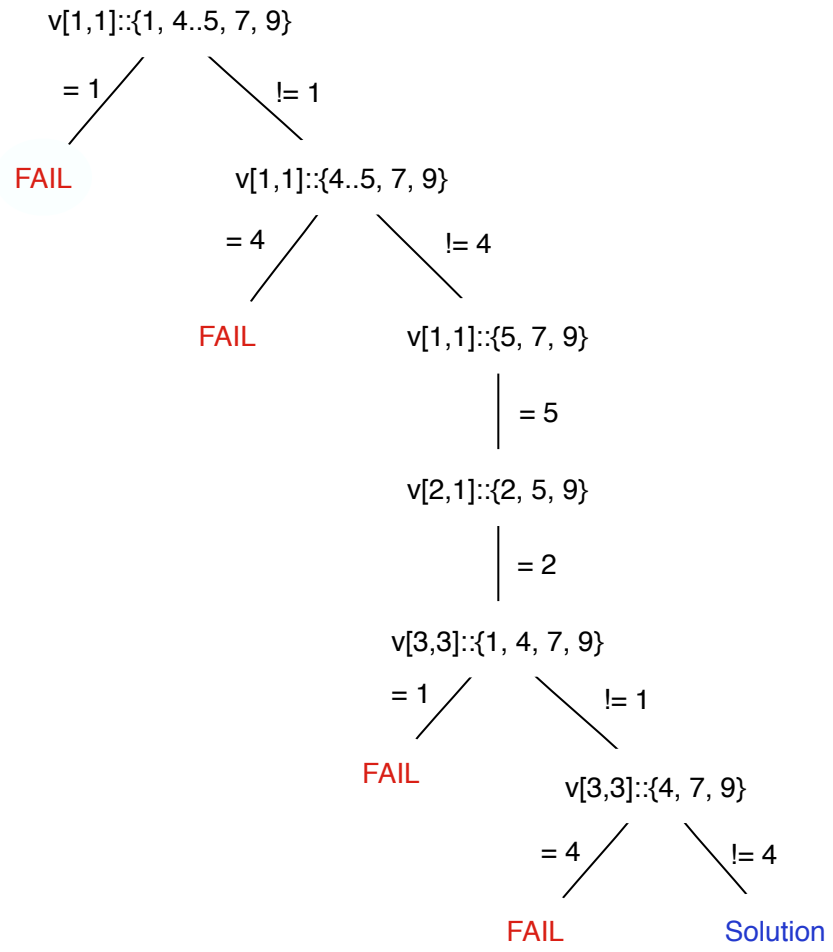
```
include "globals.mzn";

array [1..9, 1..9] of var 1..9: v;

predicate row_diff(int: r) =
    all_different ([v[r, c] | c in 1..9]);
predicate col_diff(int: c) =
    all_different ([v[r, c] | r in 1..9]);
predicate subgrid_diff(int: r, int: c) =
    all_different ([v[r + i, c + j] | i, j in 0..2]);

constraint forall (r in 1..9)         (row_diff(r));
constraint forall (c in 1..9)         (col_diff(c));
constraint forall (r, c in {1, 4, 7}) (subgrid_diff(r, c));

solve satisfy;

output ["v = ", show(v), "\n"];

v = [| 2, 6, _, 3, _, _, _, _, _
     | 5, _, _, _, _, _, 7, _, _
     | _, _, _, _, _, 1, _, 4, _
     | 6, _, _, 5, _, _, 2, _, _
     | _, _, _, _, _, _, _, _, _
     | _, _, 4, _, _, 8, _, _, 1
     | _, 5, _, 9, _, _, _, _, _
     | _, _, 7, _, _, _, _, _, 3
     | _, _, _, _, _, 4, _, 1, 6 |];
```

Another SUDOKU example

	6	8	4		1		7	
				8	5	3		
	2	6	8		9		4	7
		7				9		
	5		1		6	2		3
	4		6	1				
	3		2		7	6	9	

Another SUDOKU example

	6	8	4		1		7	
				8	5	3		
	2	6	8		9		4	7
		7				9		
	5		1		6	2		3
	4		6	1				
	3		2		7	6	9	



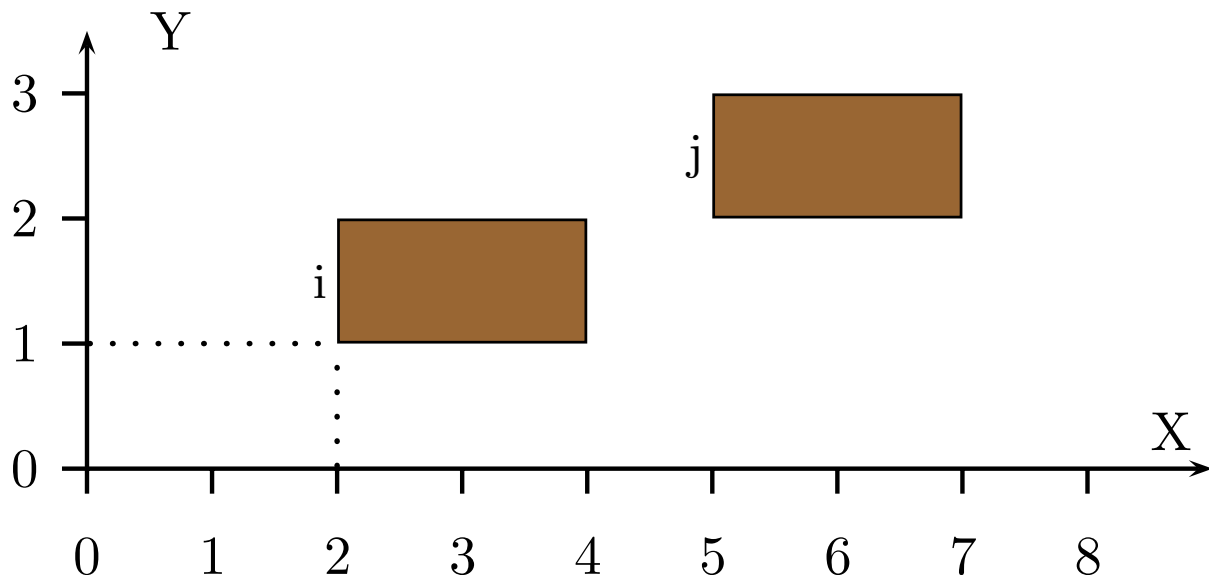
Pure JaCoP model

```
     :
Store store = new Store();

IntVar[] elements = new IntVar[noRows * noColumns][noRows * noColumns];

   // Creating variables.
   for (int i = 0; i < noRows * noColumns; i++)
      for (int j = 0; j < noRows * noColumns; j++)
         if (description[i][j] == 0)
            elements[i][j] = new IntVar(store, "f" + i + j, 1, 9);
         else
            elements[i][j] = new IntVar(store, "f" + i + j, description[i][j], description[i][j]);

    // Creating constraints for rows.
    for (int i = 0; i < 9; i++)
       store.impose(new Alldistinct(elements[i]));

    // Creating constraints for columns.
          :
    // Creating constraints for blocks.
          :
```

Constraint programming summary

- Declarative programming style
- Different domains (finite, set, interval, real, etc.)
- Primitive and global constraints (encapsulate complex pruning algorithms)
- Program has two parts:
  - variable and constraints
  - search
- Complete and heuristic search methods

Global Constraints- Diff2



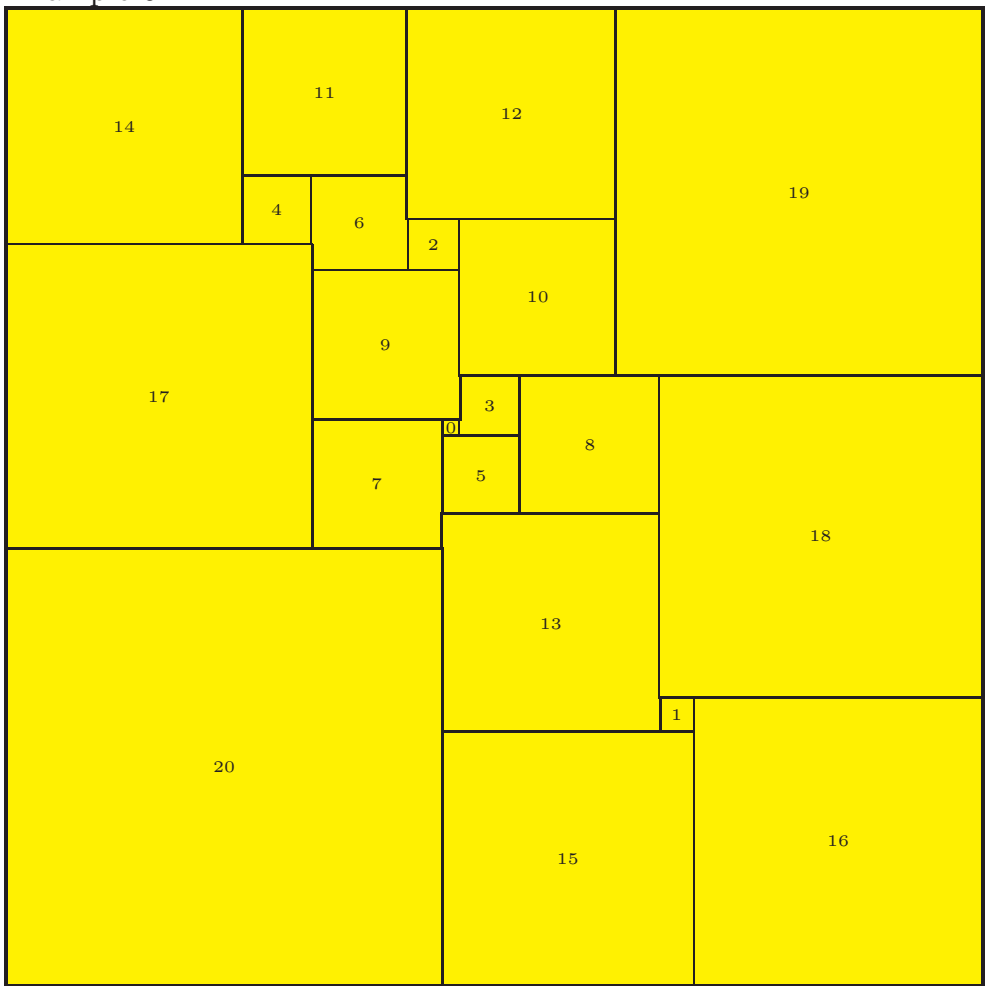
$\texttt{diff2}([[x_1, y_1, dx_1, dy_1], [x_2, y_2, dx_2, dy_2]], \ldots)$

Perfect Square Example

```
include "diff2.mzn";

int: n;                          % number squares
int: size;                       % size of the square to fill with n squares
array[1..n] of int: squares;     % dimension of the squares

array[1..n] of var 0..size: x; % x position of squares
array[1..n] of var 0..size: y; % y position of squares

constraint
   forall(i in 1..n) (
      x[i] <= size - squares[i] /\
      y[i] <= size - squares[i] ) /\
      diff2(x, y, squares, squares);

solve :: seq_search([
                     int_search(x, smallest, indomain_min, complete),
                     int_search(y, smallest, indomain_min, complete)
         ]) satisfy;

output[show(x),show(y)];

n=21;
size=112;
squares = [2,4,6,7,8,9,11,15,16,17,18,19,24,25,27,29,33,35,37,42,50];
```

Result

Example 0



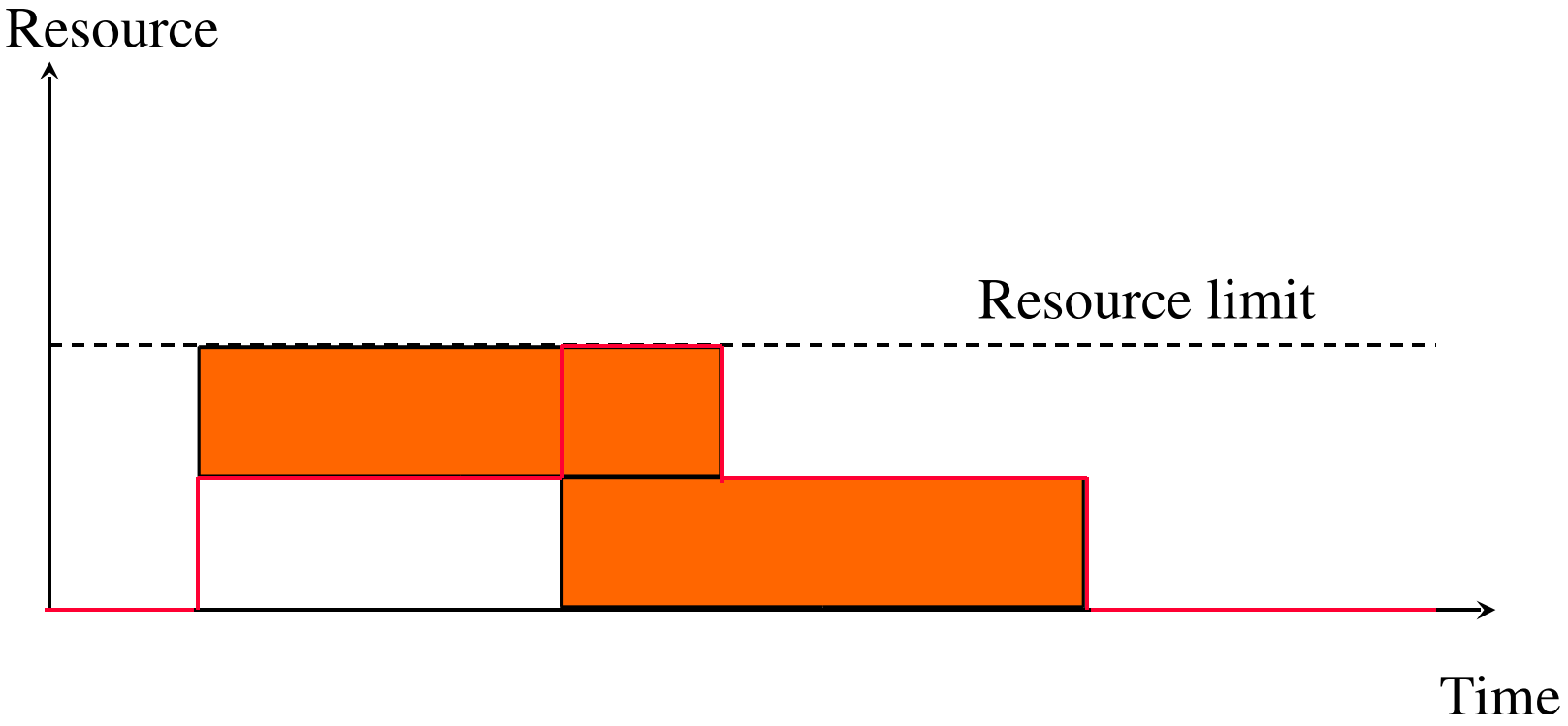
Run time: 380 ms

Scheduling

Jobshop scheduling

- Set of n jobs and m machines,
- Each job i has m ordered tasks
- Each task has constant duration and a machine for its execution,
- Find a shortest schedule.

Cumulative constraint

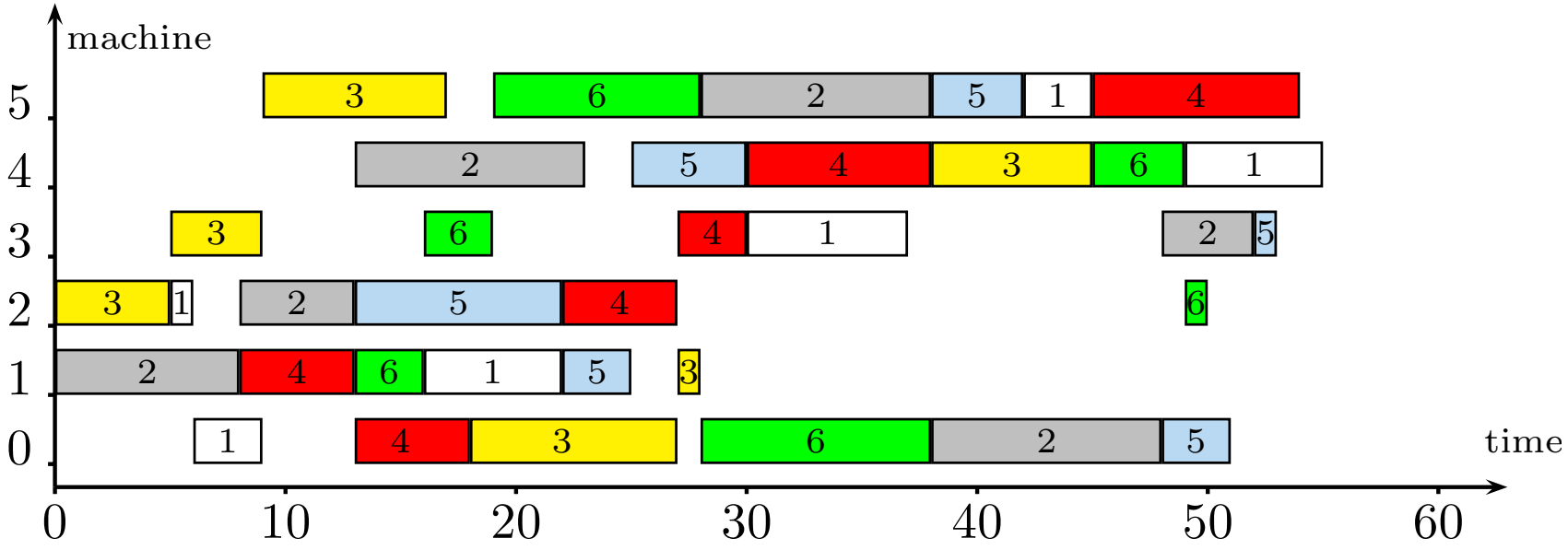


$$\texttt{cumulative}([s_1, \ldots s_n], [d_1, \ldots d_n], [r_1, \ldots r_n], \mathit{Limit})$$

Jobshop model

```
include "globals.mzn";

int: n;
int: m;
array [1..n, 1..2*m] of int: job;
array [1..n, 1..m] of var 0..1000: t :: is_output;
array [1..n] of int: one = [ 1 | i in 1..n];
var 0..1000: end :: is_output;

constraint
   forall(i in 1..n, j in 1..m-1) ( t[i,j] + job[i, 2*j] <= t[i, j+1] );

constraint
   forall(k in 1..m) (
   let {
      array[1..n] of int: d = [ job[i, 2*j] | i in 1..n, j in 1..m where job[i,2*j-1] = k-1],
      array[1..n] of var 0..1000: s = [ t[i,j] | i in 1..n, j in 1..m where job[i,2*j-1] = k-1]
   }
   in
     cumulative(s, d, one, 1)
   );

constraint
   end = max(i in 1..n) (t[i, m] + job[i, 2*m]);

solve :: int_search([ t[i,j] | i in 1..n, j in 1..m], smallest, indomain_min, complete) minimize end;
```

Jobshop Scheduling



Gantt diagram for job-shop scheduling.

Set Constraints

- Set variables, e.g. `s :: {{}..{1..5}}`
- Set operations, such as intersection, union, etc.

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Social golfers problem

A club has a number of golfers that play rounds in groups (the number of golfers is a multiple of the number of groups). Each round, a golfer plays with a group of different people, such that the same pair of golfers never play together twice.

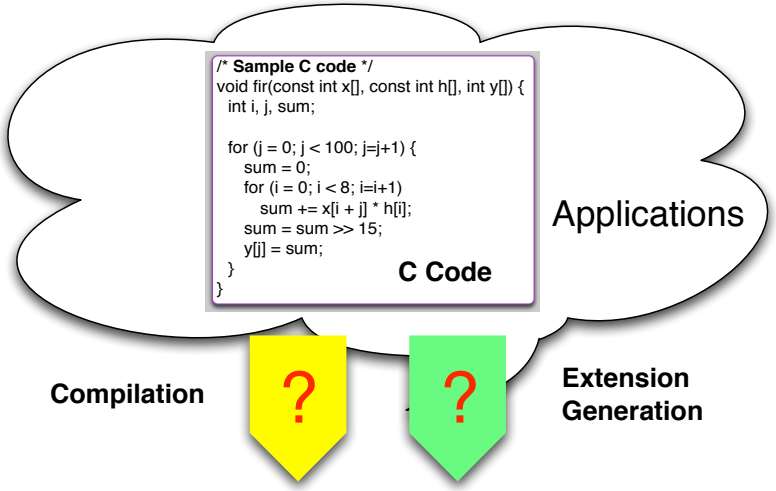
Social golfers model

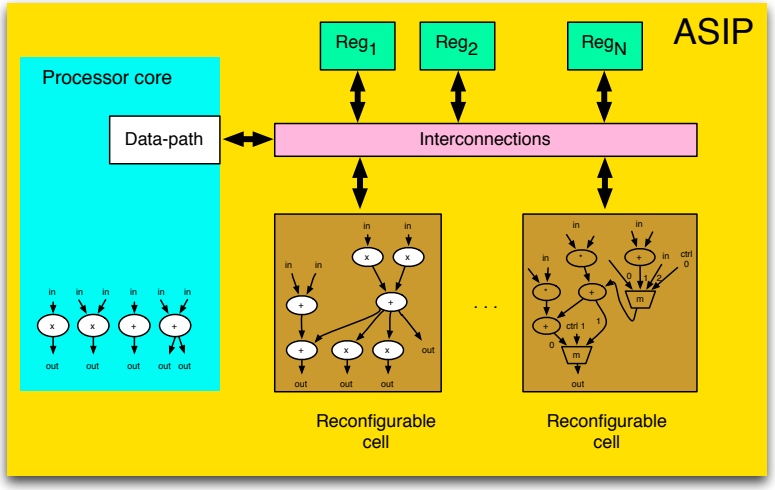
```
include "globals.mzn";

int: n_groups;                          % The number of groups.
int: n_per_group;                       % The size of each group.
int: n_rounds;                          % The number of rounds.
int: n_golfers = n_groups * n_per_group;

set of int: groups = 1..n_groups;
set of int: group = 1..n_per_group;
set of int: rounds = 1..n_rounds;
set of int: golfers = 1..n_golfers;

array [rounds, groups] of var set of golfers: round_group_golfers :: is_output;

constraint    % Each group has to have the right size.
    forall (r in rounds, g in groups) (
        card(round_group_golfers[r, g]) = n_per_group );

constraint    % Each group in each round has to be disjoint.
    forall (r in rounds) (
        all_disjoint (g in groups) (round_group_golfers[r, g]) );

constraint    % Each pair may play together at most once.
    forall (a, b in golfers where a < b) (
        sum (r in rounds, g in groups) (
            bool2int({a, b} subset round_group_golfers[r, g]) )  <=  1 );

solve satisfy;
```

A Solution

```
Groups = 4, Size = 4, Round = 5

 {1..4},          {5..8},          {9..12},         {13..16},
 {1, 5, 9, 13},   {2, 6, 10, 14},  {3, 7, 11, 15},  {4, 8, 12, 16},
 {1, 6, 11, 16},  {2, 5, 12, 15},  {3, 8..9, 14},   {4, 7, 10, 13},
 {1, 7, 12, 14},  {2, 8, 11, 13},  {3, 5, 10, 16},  {4, 6, 9, 15},
 {1, 8, 10, 15},  {2, 7, 9, 16},   {3, 6, 12..13},  {4..5, 11, 14}
```

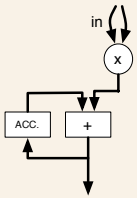
Project


/* Sample C code */
void fir(const int x[], const int h[], int y[]) {
int i, j, sum;
for (j = 0; j < 100; j=j+1) {
sum = 0;
for (i = 0; i < 8; i=i+1)
sum += x[i + j] * h[i];
sum = sum >> 15;
y[j] = sum;
}
}
C Code
Applications
Compilation
?
?
Extension Generation


ASIP
Reg1
Reg2
RegN
Processor core
Data-path
Interconnections
Reconfigurable cell
Reconfigurable cell

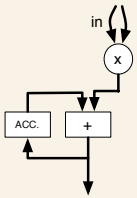
Idea: Compilation Principle

DSP Data Path

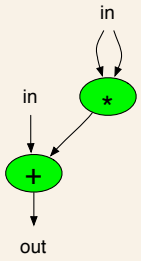


Idea: Compilation Principle

DSP Data Path



DFG "Pattern" of MAC



Idea: Compilation Principle

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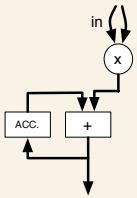

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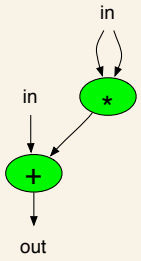
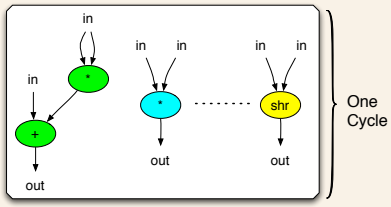
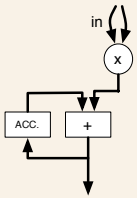


DSP Instruction Set



Idea: Compilation Principle

DSP Data Path



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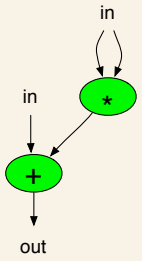
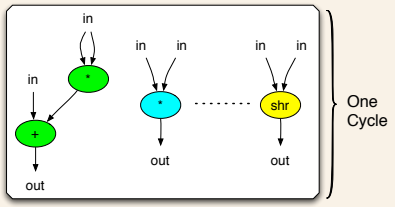
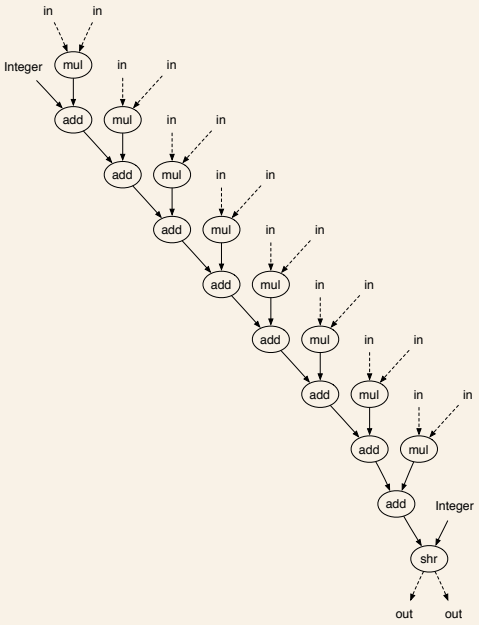


DSP Instruction Set

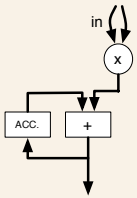


DFG of the FIR filter

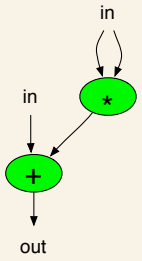


Idea: Compilation Principle

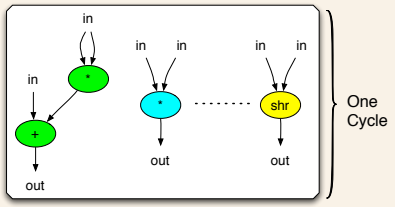
DSP Data Path



DFG "Pattern" of MAC



DSP Instruction Set



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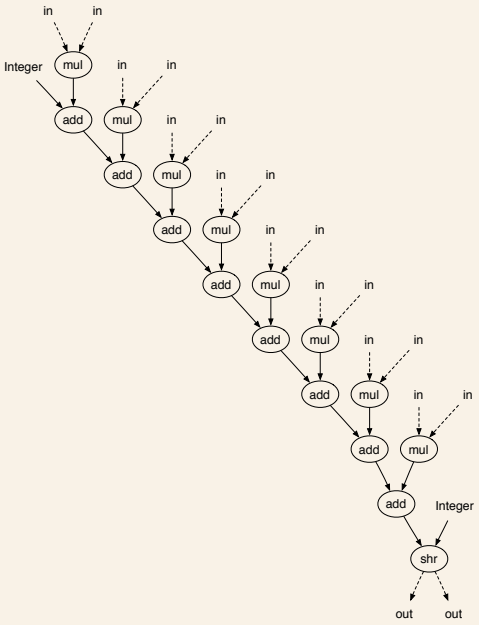
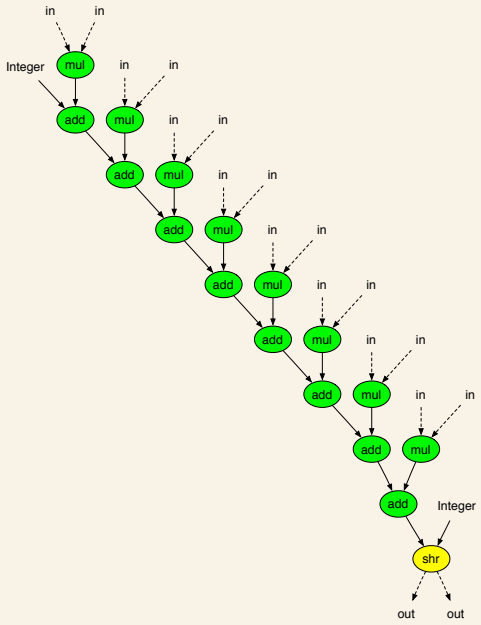
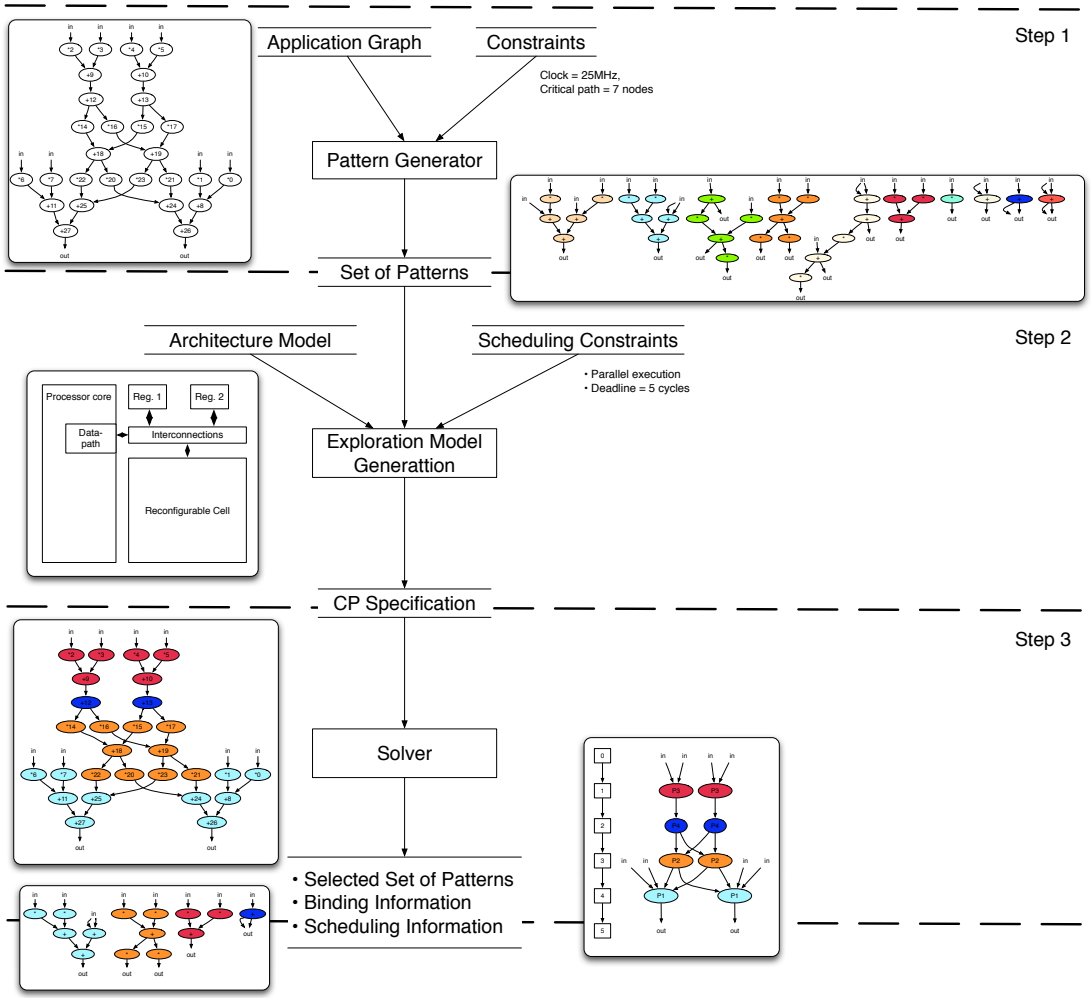


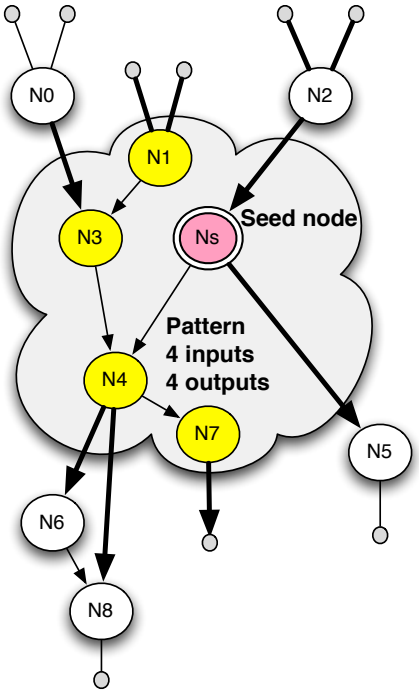
Covered DFG



Design Flow


Step 1
Application Graph
Constraints
Clock = 25MHz,
Critical path = 7 nodes
Pattern Generator
Set of Patterns
Step 2
Architecture Model
Scheduling Constraints
• Parallel execution
• Deadline = 5 cycles
Processor core
Reg. 1
Reg. 2
Data-path
Interconnections
Reconfigurable Cell
Exploration Model Generattion
CP Specification
Step 3
Solver
• Selected Set of Patterns
• Binding Information
• Scheduling Information

Idea: Pattern Generation Principle

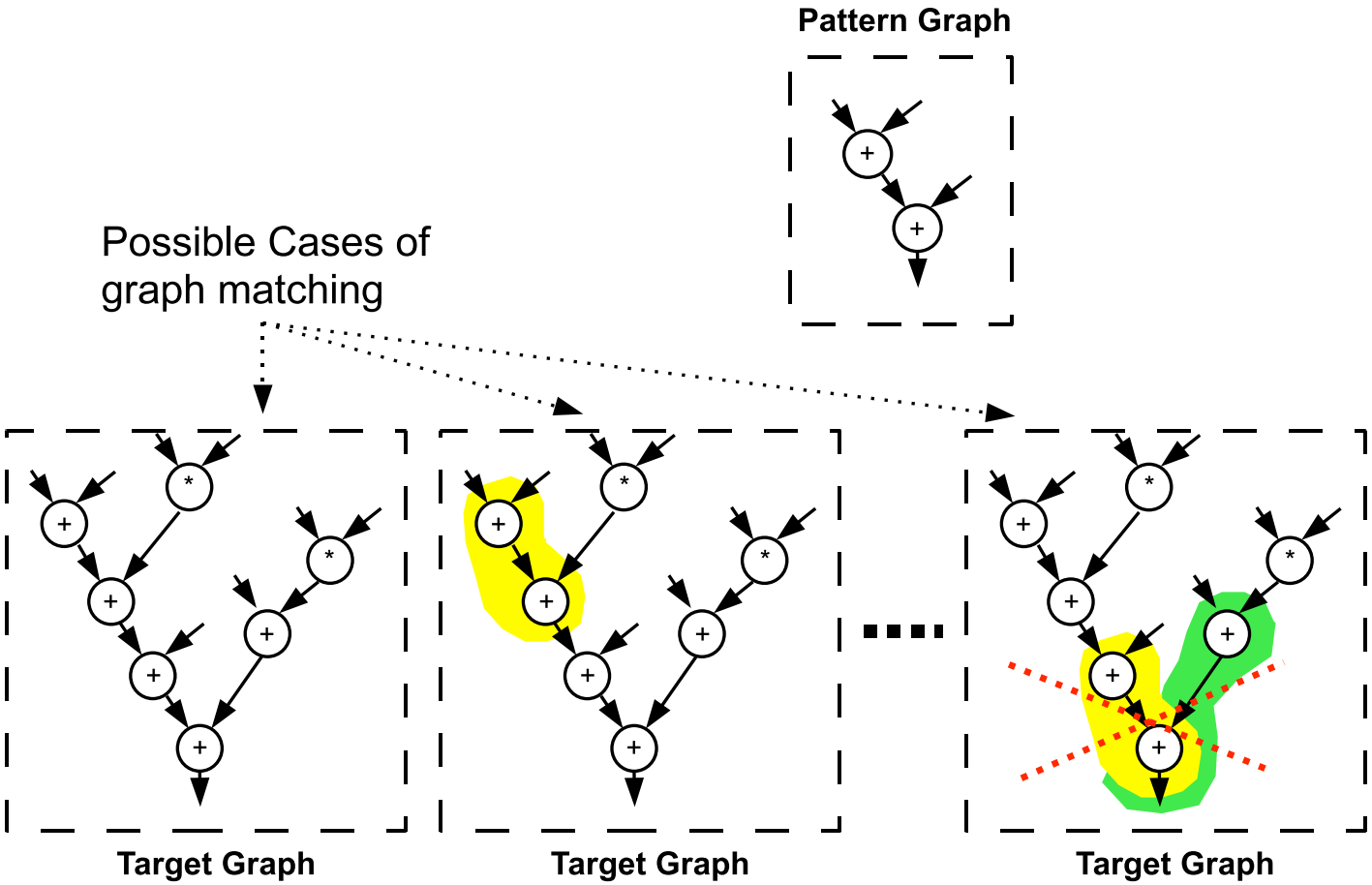


Generate patterns for all $n_s \in N_{ApplicationGraph}$

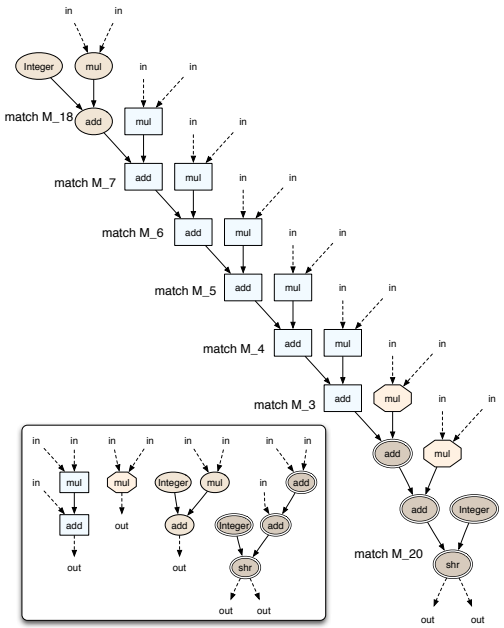
- Connected component for node n_s
- Additional constraints
  - number of inputs $\leq$ *PatternInputs*
  - number of outputs $\leq$ *PatternOutputs*
  - critical path $\leq$ *PatternCriticalPath*
  - number of nodes $\leq$ *PatternNumberOfNodes*
  - power consumption $\leq$ *PatternPowerConsumption*
  - ...
- Reduce number of patterns (remove isomorphic patterns, add most frequent patterns, etc.)

(Sub-)graph isomorphism

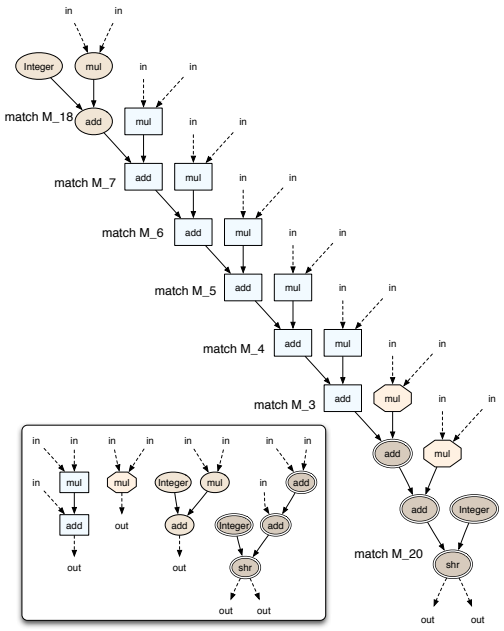
$\texttt{GraphMatch}(Graph_{Target}, Graph_{Pattern})$

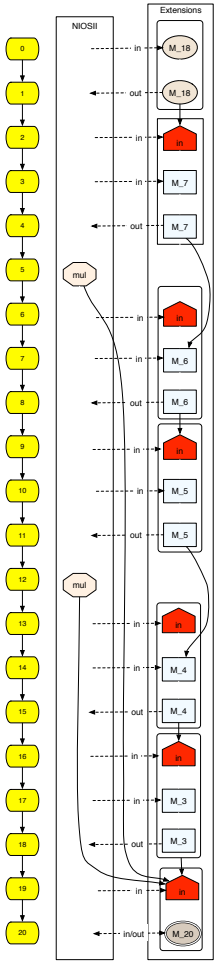


Scheduling Example - FIR filter

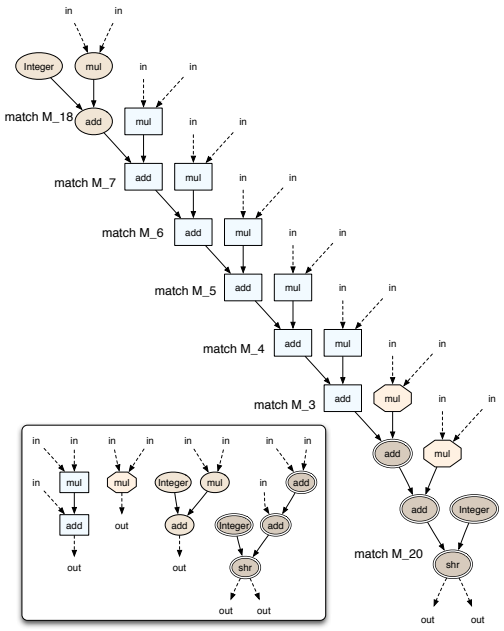


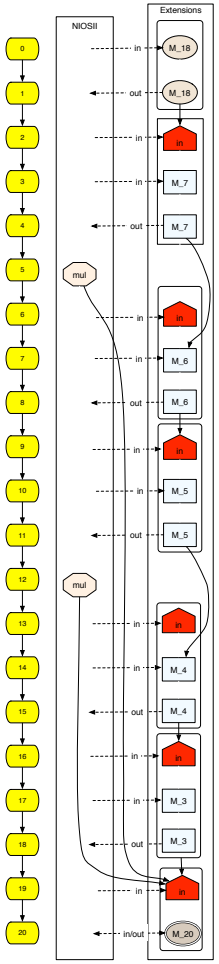
Scheduling Example - FIR filter

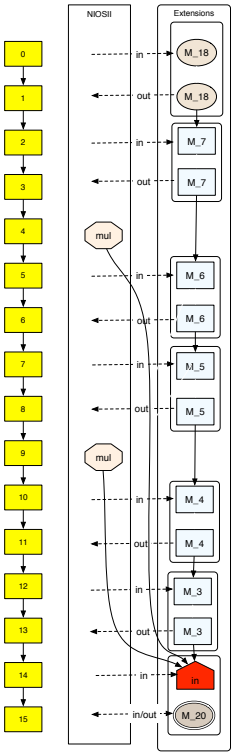




Scheduling Example - FIR filter







Results

Table: Results obtained for sequential scheduling and parallel scheduling.

			Sequential scheduling				Parallel scheduling (model A)							Parallel scheduling (model B)					
Benchmarks	|V|	Cycles	Selected	Cycles	Speedup	Runtime (s)	Selected	Cycles	Parallel memory accesses	Local variables	Number cells	Speedup S_1	Runtime (s)	Selected	Cycles	Number cells	Speedup S_2	Runtime (s)	Percent of S_1
JPEG IDCT	200	254	6	195	1.3	18.9	28	22	13	78	8	11.55	14.2	28	34	5	7.47	10.2	64%
BF	330	340	9	163	2.09	0.1	7	81	3	152	4	4.2	13.2	7	98	4	3.47	0,01	82%
MESA invert matrix	278	334	7	148	2.26	3.8	9	25	38	134	8	13.36	0.6	9	78	4	4.28	0.5	32%
with mem. access ≤ 10							9	29	10	134	8	11.52	1.99						
MCRYPT cast128	424	464	29	240	1.83	50.5	18	202	8	155	6	2.3	6.6	18	219	3	2.12	15.9	92%
GSMenc	387	433	8	117	3.7	0.1	9	16	15	132	8	27.06	6.0	9	68	2	6.37	7.2	23%
POLARSSL aes	1350	1658	12	316	2.28	6.2	15	280	27	739	8	5.92	20.2	10	482	5	3.44	43.6	58%
POLARSSL des3	398	530	15	295	1.76	5.2	26	95	6	156	5	5.58	23.9	25	109	4	4.86	24.8	87%
Maximal Speedup					3.7							27.06					7.47		
Average Speedup					2.17							9.98					4.57		

Other constraints

- Global constraints (element, regular, cumulative, geost, gcc, knapsack, extensional_support, etc.)
- More set constraints
- Complex search strategies
- To try more `http://www.jacop.eu` (MiniZinc `http://www.g12.cs.mu.oz.au/minizinc/`)

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Selected Publications

K. Kuchcinski.
Constraints-driven scheduling and resource assignment.
ACM Transactions on Design Automation of Electronic Systems (TODAES), 8(3):355–383, July 2003.

K. Martin, Ch. Wolinski, K. Kuchcinski, A. Floch, and F. Charot.
Constraint-driven instructions selection and application scheduling in the *DURASE* system.
In *20th IEEE International Conference on Application-specific Systems, Architectures and Processors (ASAP)*, Boston, USA, July7-9, 2009.

Ch. Wolinski and K. Kuchcinski.
Automatic selection of application-specific reconfigurable processor extensions.
In *Proc. Design Automation and Test in Europe*, Munich, Germany, March 10-14, 2008.

Christophe Wolinski, Krzysztof Kuchcinski, and Erwan Raffin.
Automatic design of application-specific reconfigurable processor extensions with UPaK synthesis kernel.
ACM Trans. Des. Autom. Electron. Syst., 15(1):1–36, 2009.